

Name: \_\_\_\_\_

School: \_\_\_\_\_

## Calculator Exam – UH Math Contest 2017

**Directions:** Write your name and school name on every sheet, and write your answers on the answer sheet. **DO NOT detach the answer sheet from your exam.** Answers can be given as integers, fractions or in decimal form. Answers given in decimal form should only include no more than 4 places after the decimal, and the answer should be **accurate to 4 places after the decimal.**

**DO NOT ROUND YOUR ANSWERS!!**

**Example 1:** You work on a problem, and your calculator gives the value 0.9092974268.

**Correct Answer:** 0.9092

**Incorrect Answer:** 0.9093 **DO NOT ROUND.**

**Example 2:** You determine that the answer to a question is  $1/2$ .

**Correct Answer:**  $1/2$

**Correct Answer:** 0.5

**Example 3:** You determine that the answer to a question is  $10/11$ , and your calculator tells you this has a decimal expansion 0.90909090...

**Correct Answer:**  $10/11$

**Correct Answer:** 0.9090

**Incorrect Answer:** 0.9091 **DO NOT ROUND.**

**Good Luck!!**

Name: \_\_\_\_\_

School: \_\_\_\_\_

## Calculator Exam – UH Math Contest 2017

1. Give the average of the first 17 prime numbers.
2. Give the sum of the reciprocals of the first 17 prime numbers.
3. Give the sum of the  $x$  and  $y$  values associated with points of intersection of the graphs of  $y = \sin(x)$  and  $y = \frac{1}{2}x - \frac{1}{5}x^2$ .
4. Rotate the line segment from  $(0,0)$  to  $(2,3)$  through 0.63 radians, counter clockwise, about the origin. The new segment has one endpoint at  $(0,0)$ . Give the  $y$ -coordinate of the other endpoint.
5. A number is determined from the values  $1, 2, 3, \dots, 2000$  by adding all of the values that are not integer multiples of 3, 5 or 7, and subtracting all of the values that are integer multiples of 3, 5 or 7. What is the result?
6. Give the number of points of intersection for the graphs of  $f(x) = \frac{1}{600}x^2 - \frac{13}{15}$  and  $g(x) = \sin(x)$ .
7. Give the obtuse angle of intersection (in radians) of the lines given by  $31x - 13y = 31$  and  $13x + 29y = 14$ .
8. Give the distance from the point  $(2, -3)$  to the line  $31x - 13y = 31$ .
9. The base 2 representation of a base 10 number is 1100110011. Give the number in base 10.
10.  $1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \dots + \frac{1}{1211} =$
11. The average of the first  $n$  natural numbers is 2,321,112.  
$$\frac{311n}{423n + 12} =$$

Name: \_\_\_\_\_

School: \_\_\_\_\_

## Calculator Exam – UH Math Contest 2017

12. Give the number represented graphically by the graph of the curve given parametrically by  $x = \sin(2t)$ ,  $y = \cos(t)$  for  $0 \leq t \leq 2\pi$ .

13. Give the smallest natural number  $n$  so that

$$\sin(1) + \sin\left(\frac{1}{2}\right) + \sin\left(\frac{1}{3}\right) + \cdots + \sin\left(\frac{1}{n}\right) > 9.2838$$

14. The graph of the function  $f(x) = ax^2 + bx + c$  has a root at  $x = -3.5$ , its vertex at  $x = 13.2$ , and its  $y$ -intercept at  $(0, 27.45)$ .  $a =$

15. A penny weighs 2.5 grams and a dime weighs 2.268 grams. A pile of more than 100 coins consists entirely of pennies and dimes. Give the smallest possible amount of money in the pile if the total weight of the dimes is the same as the total weight of the pennies.

16. Define  $f(x) = 3.8(x - x^2)$ , and  $g(x) = f\left(f\left(f\left(f(x)\right)\right)\right)$ . Let  $x_0 = \frac{1}{2}$  and define  $x_n = g(x_{n-1})$  for every  $n = 1, 2, 3, \dots, 100$ .  $x_{100} =$

17. A octagon centered at  $(0,0)$  has one of its vertices at  $(2,3)$ . Give the distance from  $(0,0)$  to the point on the hexagon that is closest to  $(0,0)$ .

18. Graph the curve given parametrically by  $x = (\cos(t))^{1/13}$ ,  $y = (\sin(t))^{1/13}$  for  $0 \leq t \leq 2\pi$ , and determine the  $x$ -coordinate of the point on the curve that is furthest from the origin.

19. Give the  $x$ -coordinate of the point on the graph of

$$g(x) = \frac{x^2 + 10}{3x + 1}$$

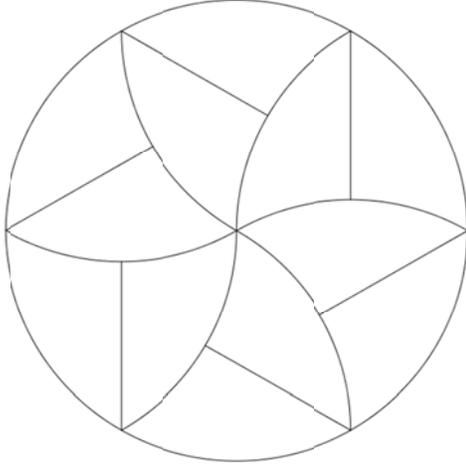
that is closest to the origin.

Name: \_\_\_\_\_

School: \_\_\_\_\_

## Calculator Exam – UH Math Contest 2017

20. The image below shows a circle of radius 1 cut into 12 congruent pieces. Give the sum of the circumferences of the 12 pieces.



21. The coordinates of the point  $(a, b, c)$  satisfy the system

$$23a + 31b + 53c = 0$$

$$-11a + 12b + 14c = 0$$

$$a^2 + b^2 + c^2 = 1$$

$$a > 0$$

$$a + b + c =$$

22. Suppose  $b > 0$ . One excellent method for approximating  $\sqrt{b}$  works as follows. Pick any value  $a_0 > 0$ . Define  $a_1$  to be the average of  $a_0$  and  $\frac{b}{a_0}$ , and in general, for each natural number  $n$ , define  $a_n$  to be the average of  $a_{n-1}$  and  $\frac{b}{a_{n-1}}$ . Suppose this method is used to approximate  $\sqrt{999,999,997}$  by using  $a_0 = 1$ . Give the smallest natural number  $n$  so that  $|999,999,997 - (a_n)^2| < 10^{-5}$ .

23. Suppose  $f(x)$  is a given function. A number  $a$  is said to be a **period 4 point** for  $f(x)$  if and only if  $f(a) \neq a$ ,  $f(f(a)) \neq a$ ,  $f(f(f(a))) \neq a$ , and  $f(f(f(f(a)))) = a$ . Give the sum of the  $x$ -coordinates for the period 4 points of  $f(x) = 3.8(x - x^2)$ .

Name: \_\_\_\_\_

School: \_\_\_\_\_

### Calculator Exam – UH Math Contest 2017

24. Set  $x_0 = 1$ ,  $y_0 = 4$ , and let  $(x_1, y_1)$  be the solution to
- $$\begin{cases} 0.5404x_1 + 0.8414y_1 = x_0 \\ -0.8415x_1 + 0.5403y_1 = y_0 \end{cases}$$

In general, for each natural number  $n$ , let  $(x_n, y_n)$  be the solution to

$$\begin{cases} 0.5404x_n + 0.8414y_n = x_{n-1} \\ -0.8415x_n + 0.5403y_n = y_{n-1} \end{cases}$$

Give  $y_{25}$ .

25. For each natural number  $n$ , let  $\theta_n = \frac{1}{n}$ . For a given real number  $a > 0$ , let  $d_1 = 1$ , and choose

$$d_2 = \begin{cases} -1 & \text{if } \theta_1 \geq a \\ 1 & \text{if } \theta_1 < a \end{cases}$$

In general, for each natural number  $n \geq 3$ , choose

$$d_n = \begin{cases} -1 & \text{if } d_1\theta_1 + \cdots + d_{n-1}\theta_{n-1} \geq a \\ 1 & \text{if } d_1\theta_1 + \cdots + d_{n-1}\theta_{n-1} < a \end{cases}$$

Let  $a = 3$ .  $d_1\theta_1 + \cdots + d_{1,000}\theta_{1,000} =$