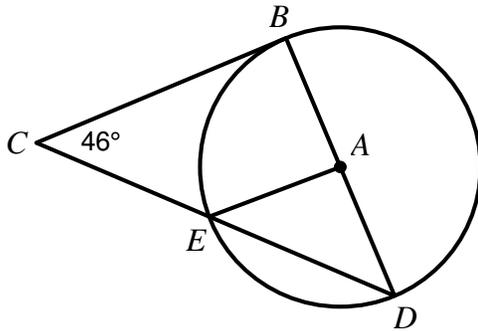


## University of Houston Mathematics Contest Geometry Exam – Spring 2017

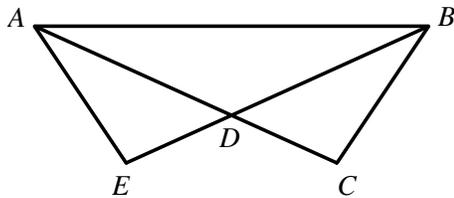
Answer the following. Note that diagrams may not be drawn to scale.

1. In the figure below,  $\overline{CB}$  is tangent to circle A. Find the measure of  $\angle BAE$ .



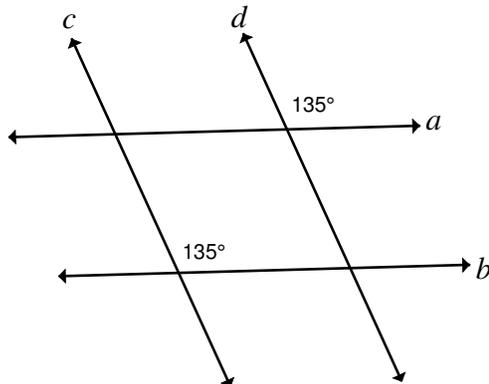
- A.  $88^\circ$
- B.  $92^\circ$
- C.  $46^\circ$
- D.  $90^\circ$
- E.  $44^\circ$
- F. None of these

2. In the diagram below,  $\overline{AC} \cong \overline{BE}$  and  $\angle ABE \cong \angle BAC$ . Which theorem or postulate can be used to prove that  $\triangle ABE \cong \triangle BAC$ ?



- A. Angle-Angle-Side
- B. Side-Side-Side
- C. Angle-Side-Angle
- D. Angle-Side-Side
- E. Side-Angle-Side
- F. None of these

3. Which lines are parallel in the diagram, based on the angle measures shown?



- A.  $a \parallel b$  only
- B.  $c \parallel d$  only
- C.  $a \parallel b$  and  $c \parallel d$
- D. No conclusion can be drawn.

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4. Given the following conditional statement:

*If a polygon is equiangular, then it is a regular polygon.*

Determine the truth value of the inverse, contrapositive, and converse, respectively.

- A. *False, True, False*      B. *True, False, True*      C. *True, True, True*  
D. *False, False, False*      E. *False, False, True*      F. None of these

5. Consider the word shown below.

S P I D E R M A N

Let  $x$  represent the number of letters in the above word which are rotationally symmetric, and let  $y$  represent the number of letters in the above word which are reflectionally symmetric. Find the value of  $x + y$ .

- A. 6      B. 8      C. 10      D. 7      E. 9      F. None of these
6. Each student in Mrs. Miller's geometry class is given a square pyramid made of styrofoam, along with a tool to slice it. Mrs. Miller instructs each student to slice their pyramid with a plane in such a way that the plane is perpendicular to two of the pyramid's base edges. Each student then displays their result to their classmates and reports the cross-sectional shape that was formed. Which list below represents all possible cross-sections that the class could obtain after following Mrs. Miller's instructions correctly?
- A. Triangle  
B. Square  
C. Trapezoid  
D. Triangle, Square  
E. Trapezoid, Triangle  
F. Triangle, Trapezoid, Square

7. If the diagonals of a quadrilateral are perpendicular, then the quadrilateral is a \_\_\_\_\_.

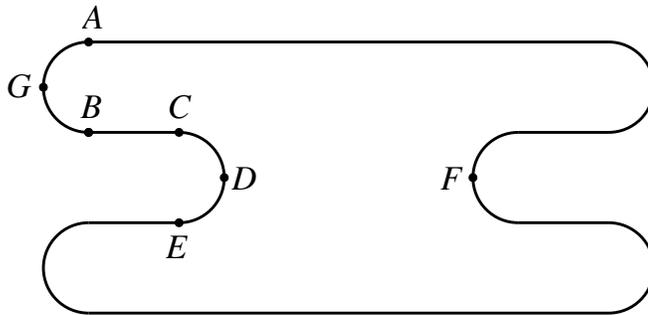
- I. kite      II. square      III. rhombus
- A. I only      B. III only      C. I and II only  
D. I and III only      E. I, II, and III      F. None of these

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8. One side of a triangle measures 8.8 cm and another side measures 13.9 cm. If the length of the third side is a whole number, how many side lengths are possible for the third side?

A. 22      B. 19      C. 18      D. 17      E. 16      F. None of these

9. A bandage company makes a bandage in the shape below which is designed to wrap around people's knuckles. The bandage is rotationally and reflectionally symmetric, with  $\overline{DF}$  being a line of symmetry.  $\widehat{AGB}$  and  $\widehat{CDE}$  are semicircles, each with arclength  $6\pi$  cm. If  $DF = 27$  cm, and  $BC = CE$ , find the area of the bandage in square centimeters.



A.  $1980 + 72\pi$       B.  $2268 + 36\pi$       C.  $810 + 9\pi$   
 D.  $1980 + 36\pi$       E.  $1836 + 36\pi$       F. None of these

10. Given the following information about a diagram: (Diagram not shown.)

$$\begin{array}{llll} XP = 6 & PY = 3 & NX = 17 & NP = 11 \\ YT = 11 & PT = 8 & PA = AN & \end{array}$$

Which of the following statements must be true?

- I.  $N$  is between  $P$  and  $X$ .  
 II.  $A$  is the midpoint of  $\overline{PN}$ .  
 III.  $T$ ,  $Y$  and  $P$  are collinear.

A. I and II only      B. II and III only      C. III only  
 D. II only      E. I, II and III      F. None of these

11. In a convex regular polygon, the ratio of an exterior angle to the interior angle at any given vertex is 1:9. How many sides does the polygon have?

A. 18      B. 20      C. 30      D. 15      E. 10      F. None of these

12. Point  $H$  has coordinates  $(-4, -2)$  and is reflected over line  $\ell$ , resulting in the image  $H'$  with coordinates  $(2, 5)$ . Find the equation of line  $\ell$ .

A.  $y = -\frac{6}{7}x + \frac{85}{14}$

B.  $y = \frac{7}{6}x + \frac{8}{3}$

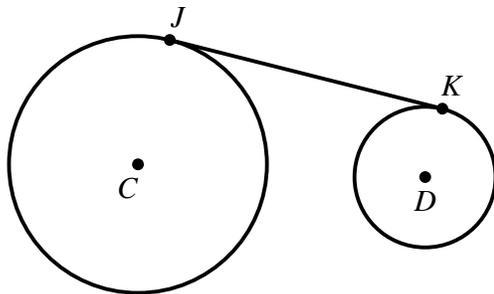
C.  $y = -\frac{7}{6}x + \frac{1}{3}$

D.  $y = \frac{6}{7}x + \frac{33}{14}$

E.  $y = -\frac{6}{7}x + \frac{2}{7}$

F. None of these

13. In the figure below,  $\overline{JK}$  is tangent to circles  $C$  and  $D$ . The diameter of circle  $C$  is 16 cm, the diameter of circle  $D$  is 6 cm, and the length of  $\overline{JK}$  is 10 cm. Find the distance between the centers of the circles.



A.  $10\sqrt{2}$  cm

B.  $\sqrt{109}$  cm

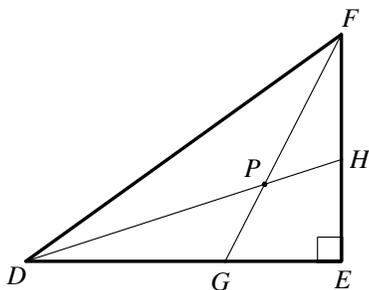
C. 10 cm

D.  $2\sqrt{82}$  cm

E.  $5\sqrt{5}$  cm

F. None of these

14. Right triangle  $DEF$  is given below, with angle bisectors  $\overline{FG}$  and  $\overline{DH}$  intersecting at  $P$ . If  $m\angle GPH = x$  and  $m\angle GPD = y$ , find the ratio  $x : y$ .



A. 3:2

B. 5:3

C. 2:1

D. 3:1

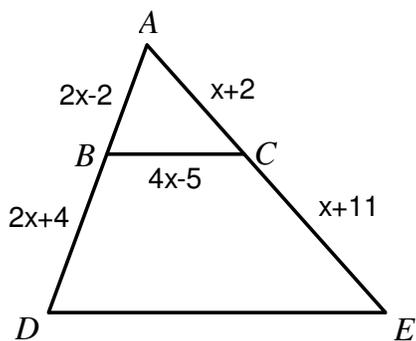
E. Cannot be determined

F. None of these

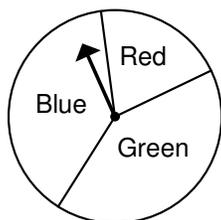
15. A sector of a circle has arclength  $7\pi$  cm and radius 10 cm. Find the central angle of the sector in radians.

A.  $\frac{20\pi}{7}$     B.  $\frac{7\pi}{10}$     C.  $70\pi$     D.  $\frac{10}{7\pi}$     E.  $\frac{7}{20}$     F. None of these

16. In the diagram below,  $\overline{BC} \parallel \overline{DE}$ . Use the given measurements to find the length of  $\overline{DE}$ .



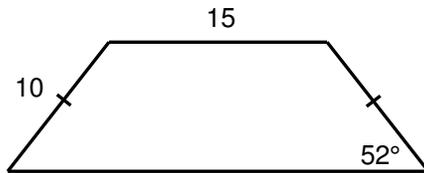
- A. 15    B. 8    C. 20    D. 17    E. 10    F. None of these
17. The spinner below is divided into three unequal sectors which display the colors red, blue, and green. The radius of the spinner measures 12 cm, the arclength of the blue sector is  $9\pi$  cm, and the area of the red sector is  $30\pi$  cm<sup>2</sup>. If the spinner is spun twice, find the probability that both spins are green.



- A.  $\frac{5}{6}$     B.  $\frac{25}{576}$     C.  $\frac{5}{33}$     D.  $\frac{25}{144}$     E.  $\frac{9}{64}$     F. None of these
18. In spherical geometry, how many equilateral right triangles are needed to cover the surface of a sphere?
- A. 6  
 B. 8  
 C. 12  
 D. 36  
 E. Equilateral right triangles do not exist.  
 F. None of these

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19. An isosceles trapezoid is shown below. If  $m = \sin(52^\circ)$  and  $n = \cos(52^\circ)$ , which of the following expressions represents the area of the isosceles trapezoid?



- A.  $\frac{150n + 100}{mn}$       B.  $150m + 100mn$       C.  $250mn$   
D.  $150n + 100mn$       E.  $\frac{150m + 100}{mn}$       F. None of these
20. Tom and Jerry go to a special candle-making shop to create custom candles. Each of them makes their own cylindrically-shaped wax candle, and each candle contains the same amount of wax. Tom's candle is three times taller than Jerry's candle. Let  $L.A._{Tom}$  and  $L.A._{Jerry}$  represent the lateral areas of Tom and Jerry's candles, respectively. Find  $L.A._{Tom} : L.A._{Jerry}$ .
- A. 1:1  
B.  $\sqrt{3} : 3$   
C. 3:1  
D.  $\sqrt{3} : 1$   
E. 9:1  
F. None of these
21. The area of a convex regular polygon is  $192\sqrt{3}$  cm<sup>2</sup>. The polygon has nine diagonals. Find the perimeter of the polygon, in centimeters.
- A.  $24\sqrt{2}$   
B. 48  
C. 96  
D.  $48\sqrt{2}$   
E.  $36\sqrt{3}$   
F. None of these

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22. Consider a circle with a diameter of length 10. Point  $A$  lies on the circle and has coordinates  $(-1, 11)$ . Find the equation of the circle, given that diameter  $\overline{AB}$  lies entirely in the second quadrant and has slope  $\frac{4}{3}$ .

A.  $x^2 + y^2 + 8x - 14y + 40 = 0$

B.  $x^2 + y^2 + 5x - 18y + \frac{249}{4} = 0$

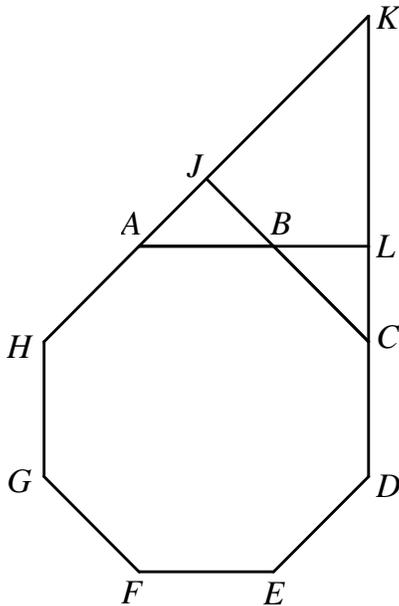
C.  $x^2 + y^2 + 10x - 16y + 64 = 0$

D.  $x^2 + y^2 + 8x - 14y - 35 = 0$

E.  $x^2 + y^2 + 14x - 6y + 33 = 0$

F. None of these

23. Regular octagon  $ABCDEFGH$  is shown below.  $J$ ,  $K$ , and  $L$  represent points of intersection when four sides of the octagon are extended. If  $DE = 12$ , find the area of quadrilateral  $JBLK$ .



A.  $288 + 144\sqrt{2}$

B.  $72 + 72\sqrt{2}$

C.  $36 + 36\sqrt{2}$

D.  $108 + 72\sqrt{2}$

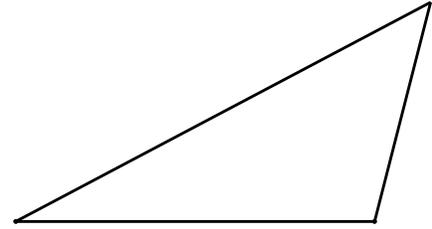
E.  $36\sqrt{3}$

F. None of these

24. Jenna is using a computer program to generate geometrical shapes and patterns.

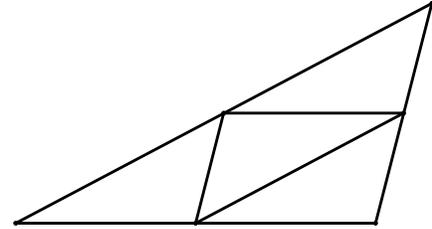
**Step 1:**

Jenna needs to create a triangle. She creates the triangle at the right.



**Step 2:**

Jenna connects the midpoints of the sides of the triangle from Step 1. This process creates four smaller triangles which are congruent to each other.



**Step 3:**

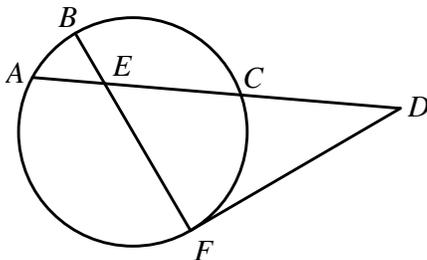
Jenna connects the midpoints of the sides of the four triangles formed in Step 2. This creates  $n$  smaller triangles which are congruent to each other.

**Step 4:**

Jenna connects the midpoints of the sides of the  $n$  triangles formed in Step 3.

Consider the set of points representing all the vertices of the triangles in Jenna's diagram after completing Step 4. If Jenna randomly selects one of these vertices, what is the probability that the chosen vertex does NOT lie on the perimeter of the original triangle (the triangle that Jenna created in Step 1)?

- A.  $\frac{1}{9}$       B.  $\frac{25}{64}$       C.  $\frac{1}{5}$       D.  $\frac{4}{15}$       E.  $\frac{7}{15}$       F. None of these
25.  $\overline{BF}$  is a diameter of the circle below, and  $\overline{DF}$  is tangent to the circle at point  $F$ . If  $AE = 3$ ,  $EF = 12$ , and  $m\angle AEF = 120^\circ$ , find the ratio  $FE : EB$ .



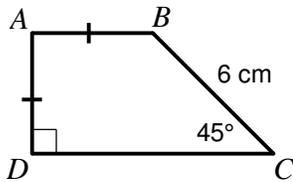
- A. 4:1      B. 3:1      C. 6:1      D. 2:1      E.  $\sqrt{3}:1$       F. None of these

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26. Given convex hexagon  $ABCDEF$ , how many triangles can be drawn whose three vertices are also vertices of the hexagon?

- A. 20
- B. 120
- C. 18
- D. 30
- E. 24
- F. None of these

27. Trapezoid  $ABCD$  is shown below. The trapezoid is rotated about side  $\overline{AD}$ , tracing out a three-dimensional solid. Find the volume of the solid that is formed, in cubic centimeters.

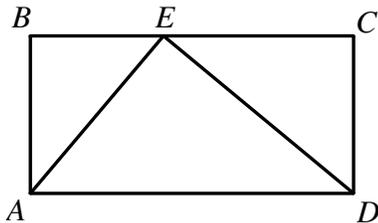


- A.  $378\pi\sqrt{2}$
- B.  $72\pi\sqrt{2}$
- C.  $126\pi\sqrt{2}$
- D.  $63\pi$
- E.  $144\pi\sqrt{2}$
- F. None of these

Questions 28-30 are write-in answers rather than multiple choice. Write the answer to each question on the answer sheet in the space provided. All irrational answers (containing radicals or  $\pi$ ) should be left as exact answers rather than decimal approximations. All radicals and fractions should be written in simplest form, with no radicals in the denominator. Do not write any units on the answer sheet.

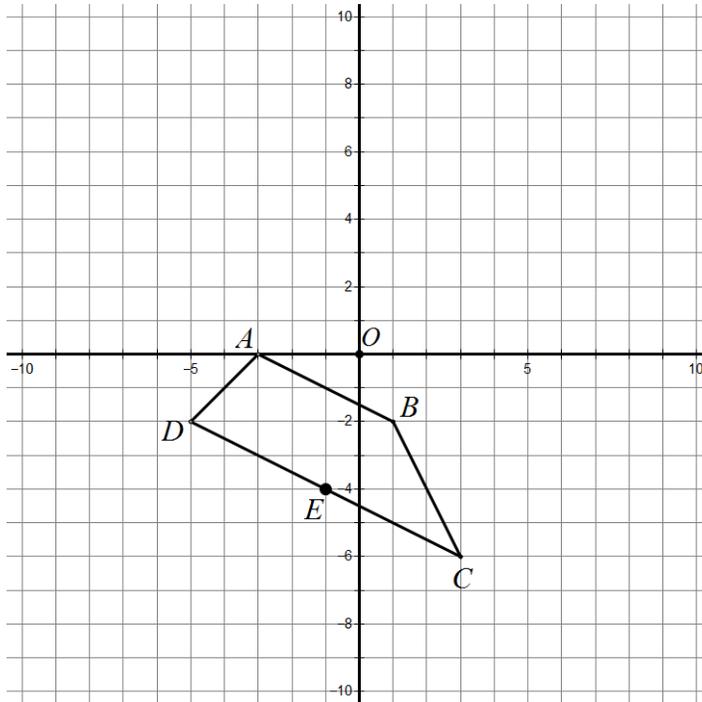
28. Rectangle  $ABCD$  is shown below. Find  $AE$ , given the following additional information:

$$EC = 4 \quad CD = 6 \quad \angle EAD \text{ and } \angle CED \text{ are complementary}$$



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29. Trapezoid  $ABCD$  is dilated using scale factor  $150\%$  and center  $E$ , resulting in trapezoid  $A'B'C'D'$ . Consider the endpoints of the median of trapezoid  $A'B'C'D'$ , and give the coordinates of the endpoint that is closest to the origin,  $O$ .



30. A regular tetrahedron is a solid made up of four equilateral triangles. Suppose that each edge of a tetrahedron has length  $x$ . Give a formula for the volume of the tetrahedron in terms of  $x$ .

**END OF EXAM** 😊