University of Houston Mathematics Contest 2018

Algebra 2 Exam

Name _____ School: _____

1. Find $x + y$, where x and y satisfy the given pair of equations:

$$x - 2y = 16$$

$$x^2 - 4y^2 = 512$$

(A) 24 (B) 26 (C) 28 (D) 30 (E) 32 (F) 34

2. Solve for x:

$$\log_2(\log_{10} \sqrt[3]{x}) = -1,$$

(A) $2\sqrt{2}$ (B) $2\sqrt{10}$ (C) $5\sqrt{2}$ (D) $5\sqrt{10}$ (E) $10\sqrt{2}$ (F) $10\sqrt{10}$

3. Evaluate:

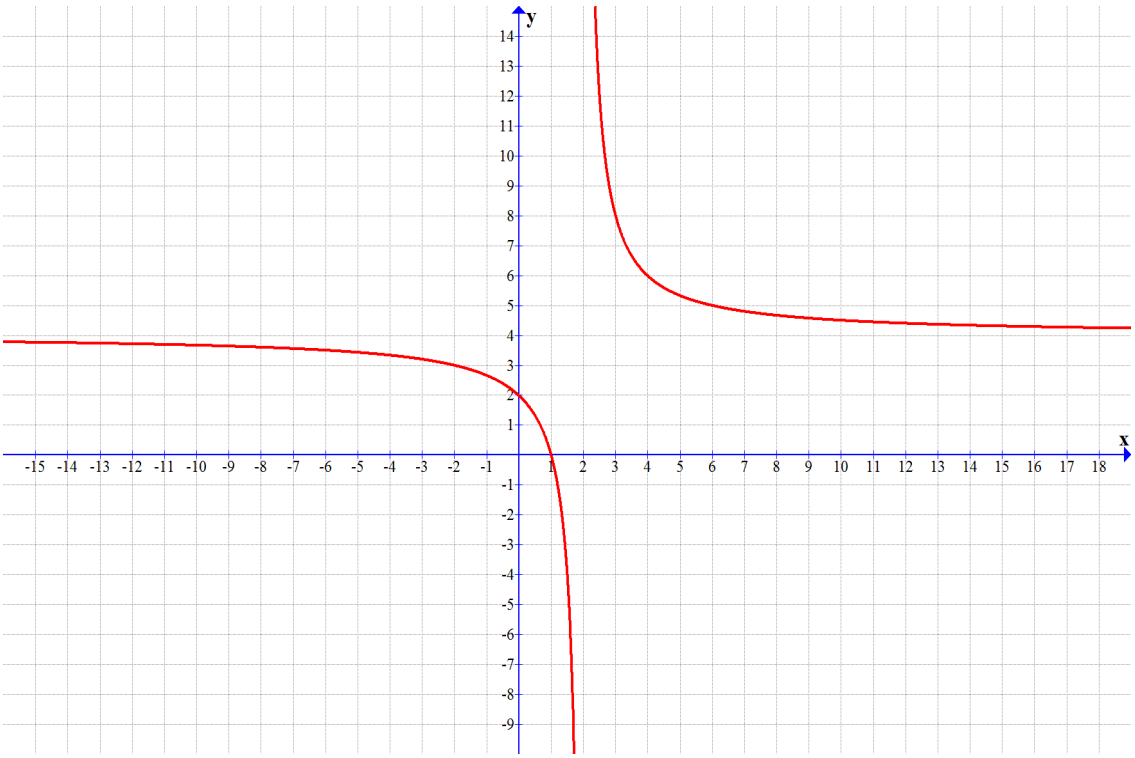
$$\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$$

(A) 1 (B) $\frac{1}{2} + \frac{\sqrt{3}}{2}i$ (C) $-\frac{\sqrt{3}}{2} + \frac{1}{2}i$

(D) i (E) $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$ (F) $\frac{\sqrt{3}}{2} - \frac{1}{2}i$

Name: _____ School: _____

4. If $f(x) = \dfrac{1}{x}$ and the graph of $af(x+b)+c$ is shown below, find the value of $a+b+c$.



(A) 1 (B) 2 (C) 3 (D) 4 (E) 5 (F) 6

5. If the expression

$$\frac{\sqrt{x\sqrt{y\sqrt{z}}}}{\sqrt{z\sqrt{x\sqrt{y}}}}$$

is written in the form $x^a y^b z^c$, find the value of $c\sqrt{b\sqrt{a}}$.

(A) $-\dfrac{1}{16}$ (B) $-\dfrac{3}{16}$ (C) $-\dfrac{1}{32}$ (D) $-\dfrac{3}{32}$ (E) $-\dfrac{1}{64}$ (F) $-\dfrac{3}{64}$

Name: _____ School: _____

6. A department store is having a clearance sale on shirts, ties, and vests. Mr. Garcia purchases 3 shirts, 2 ties, and 1 vest for 56 dollars. Mr. Smith purchases 2 shirts, 1 tie, and 3 vests for 64 dollars. Mr. Jones purchases 5 shirts, 3 ties, and 2 vests for 94 dollars. What is the price of a tie?

(A) \$9 (B) \$10 (C) \$11 (D) \$12 (E) \$13 (F) \$14

7. Find the y-coordinate of the vertex of the parabola whose graph is given by the quadratic function

$$f(x) = -3(x-2)^2 - 42(x-2) - 141.$$

(A) -9 (B) -6 (C) -3 (D) 3 (E) 6 (F) 9

8. Find the sum of all solutions of the equation

$$\frac{x+2}{x^2+3x-10} + \frac{x-5}{x^2-x-2} + \frac{x-1}{x^2+6x+5} = 0.$$

(A) -3 (B) -2 (C) -1 (D) 0 (E) 1 (F) 2

9. If x is a nonzero solution of the equation

$$x^{\frac{6}{5}} = 8x^{\frac{3}{2}},$$

find the value of $\log_2 x$.

(A) -5 (B) -6 (C) -7 (D) -8 (E) -9 (F) -10

Name: _____ School: _____

10. Consider the polynomial functions

$$p(x) = 6x^4 + 13x^3 - 31x^2 + 62x - 30$$
$$q(x) = 3x^2 - 4x + 6.$$

If the quotient $p(x)/q(x)$ is written in the form $ax^2 + bx + c$, determine the value of $a + b + c$.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5 (F) 6

11. Find the largest real number x which solves the equation

$$\sqrt[3]{x^4 + x^3 + 12x + 13} = x + 2.$$

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) $\sqrt{5}$ (E) $\sqrt{6}$ (F) $\sqrt{7}$

12. The number of students assigned to a college chemistry lab is inversely proportional to the number of instructors employed by the department. If the department employs 3 more instructors, then each lab will have 4 fewer students. If the department employs 3 fewer instructors, then each lab will have 6 more students. How many instructors are currently employed by the department.

(A) 9 (B) 12 (C) 15 (D) 18 (E) 21 (F) 24

13. If x is a positive real number that satisfies the equation

$$\log_2(x^3 + 16x^2 - 8x) = 7,$$

find the value of x.

(A) $2\sqrt{2}$ (B) 4 (C) $4\sqrt{2}$ (D) 8 (E) $8\sqrt{2}$ (F) 16

Name: _____ School: _____

14. A man takes 3 minutes to swim 100 meters upstream and 2 minutes to swim the same distance downstream. Determine the speed of the stream in meters per minute. Round your answer to the nearest integer.

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9 (F) 10

15. Find the sum of all solutions of the equation

$$f(x) = f(2x),$$

where $f(x) = x^3 - 9x^2 + 26x - 24$.

(A) $-\frac{26}{7}$ (B) $-\frac{15}{7}$ (C) $-\frac{4}{7}$ (D) $\frac{5}{7}$ (E) $\frac{16}{7}$ (F) $\frac{27}{7}$

16. The population of a small town t years from now is modeled by the function

$$P(t) = k2^t,$$

where k is a positive real number. If the population of the town is projected to be 5,120 people after 10 years, how many years will it take for the current population of the town to increase by 100 people. Express your answer in exact form.

(A) $\log_2 14$ (B) $\log_2 15$ (C) $\log_2 21$

(D) $\log_2 25$ (E) $\log_2 100$ (F) $\log_2 105$

17. Determine the number of integer-valued solutions of the inequality

$$\frac{3x + 92}{x^2 + 1} > 2.$$

(A) 9 (B) 10 (C) 11 (D) 12 (E) 13 (F) 14

Name: _____ School: _____

18. Determine the solution set of the following equation. Express your answer using interval notation.

$$\sqrt{36x^2 + 48x + 16} = 6x + 4$$

(A) $(-\infty, \infty)$ (B) $\left[-\frac{2}{3}, \frac{2}{3}\right]$ (C) $\left[-\frac{2}{3}, \infty\right)$

(D) $(-\infty, -\frac{2}{3}] \cup \left[\frac{2}{3}, \infty\right)$ (E) $(-\infty, -\frac{2}{3}]$ (F) $\emptyset$ (No solution)

19. Find xy, where x and y satisfy the given pair of equations:

$$\frac{1}{x-1} + \frac{2}{y-1} = 1$$

$$\frac{6}{x+1} - \frac{4}{y+1} = 2$$

(A) 1 (B) $\frac{5}{4}$ (C) $\frac{4}{3}$ (D) 2 (E) $\frac{5}{2}$ (F) $\frac{8}{3}$

20. If $f(x) = ax^2 + bx + c$ is a quadratic function whose graph passes through the points $(2, 5)$, $(5, -1)$, and $(-1, -25)$, find $a + b + c$.

(A) -7 (B) -4 (C) -1 (D) 2 (E) 5 (F) 8

21. If $x^2 + ax + b$ is a quadratic factor of

$$x^4 - 2x^3 - 17x^2 + 18x + 72,$$

find the smallest possible value of a.

(A) -7 (B) -6 (C) -5 (D) -4 (E) -3 (F) -2

Name: _____ School: _____

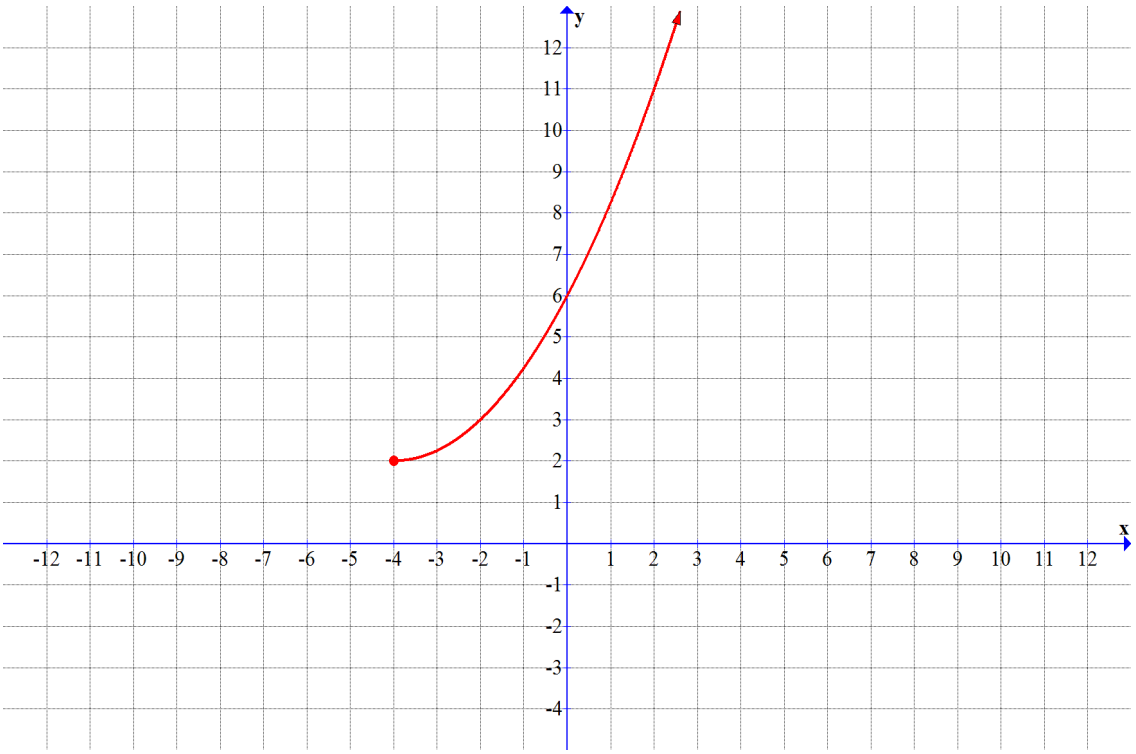
22. Assume f is an exponential function with the property that

$$f(n+1) = 8^{\frac{1}{6}} f(n),$$

for all integers n. Find the value of $\sqrt{\dfrac{f(n+3)}{f(n-5)}}$.

(A) $2\sqrt{2}$ (B) 4 (C) $4\sqrt{2}$ (D) 8 (E) $8\sqrt{2}$ (F) 16

23. If $f(x) = \sqrt{ax+b} + c$ and the graph of $f^{-1}(x)$ is shown below, find $a+b+c$.



(A) -8 (B) -4 (C) -2 (D) 2 (E) 4 (F) 8

Name: _____ School: _____

24. If the solution of the equation

$$|x-1| - 4x = 3 - |2x-1|$$

is written in the form $x = \frac{a}{b}$ where $b > 0$ and $\frac{a}{b}$ is a reduced fraction in lowest terms, find $a + b$.

(A) -9 (B) -6 (C) -3 (D) 3 (E) 6 (F) 9

25. If $\sqrt{91 - 48\sqrt{3}} = a + b\sqrt{3}$, where a and b are integers, find $a^2 + b^2$.

(A) 45 (B) 52 (C) 73 (D) 80 (E) 117 (F) 148

26. Consider the function

$$f(x) = \frac{3x-2}{x-4} + 2.$$

Which of the following functions g has the property that $(g \circ f)(x) = x$ for all $x \neq 4$?

(A) $g(x) = \dfrac{2}{x-5} + 4$

(B) $g(x) = -\dfrac{6}{x-5} + 4$

(C) $g(x) = \dfrac{10}{x-5} + 4$

(D) $g(x) = -\dfrac{14}{x-5} + 4$

(E) $g(x) = \dfrac{18}{x-5} + 4$

(F) $g(x) = -\dfrac{22}{x-5} + 4$

Name: _____ School: _____

27. Determine the range of the function

$$f(x) = \frac{x^3 - 7}{x^3 + 7}.$$

Express your answer using inteval notation.

(A) $(-\infty, \infty)$ (B) $(-\infty, -\sqrt[3]{7})$ (C) $(-\infty, -\sqrt[3]{7}) \cup (-\sqrt[3]{7}, \infty)$

(D) $(-\infty, 0)$ (E) $(-\infty, 1)$ (F) $(-\infty, 1) \cup (1, \infty)$

28. If the solution set of the inequality

$$|2x + 1| < |3x + 5| - |4x - 3|,$$

is expressed in the form $\{x \mid a < x < b\}$ where a and b are real numbers, find $a + b$.

(A) $\frac{2}{15}$ (B) $\frac{8}{15}$ (C) $\frac{14}{15}$ (D) $\frac{26}{15}$ (E) $\frac{32}{15}$ (F) $\frac{38}{15}$

29. Alan, Jim, and Stanley work together in teams of 2 or 3 on new floor installations. When Alan and Jim work together, the installation takes 16 hours, while Alan and Stanley need only 12 hours to complete the job. When all three men are working, the installation takes 9 hours to complete. If each man works at a constant rate independent of the others, how many hours would it take Alan to complete the job alone? Round your answer up to the nearest hour.

(A) 29 (B) 30 (C) 31 (D) 32 (E) 33 (F) 34

Name: _____ School: _____

30. If the ordered pair (x, y) belongs to the solution set of the system of inequalities

$$\begin{aligned} 2x - y &\geq 2 \\ x + y &\leq 7, \end{aligned}$$

which of the following inequalities is **not** always satisfied by (x, y)?

(A) $y \leq 4$ (B) $2x + 3y \leq 18$ (C) $x + 2y \leq 12$

(D) $2x - 3y \geq -6$ (E) $x - 2y \geq -4$ (F) $x - y \geq -1$