Name: ____

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Calculator Exam, Version A – 2019 UH Math Contest

Directions: Write your name and school name on every sheet. Write your answers in the table below. **DO NOT detach this sheet from your exam.** Some of the answers below are integers. The answers which are not integers should be recorded so that they are **accurate to at least 4 places *after* the decimal**. **DO NOT ROUND YOUR ANSWERS until *after* the 4th decimal place!!** For example, suppose a question requests the value of sin(2). Your calculator will tell you that sin(2) is 0.9092974268 (assuming your calculator is in radian mode). Examples of correct responses include 0.9092, 0.90929, 0.90923 and 0.90929116. **The response 0.9093 is not correct.** It does not matter what appears *after* the 4th decimal place, provided the values up to and including the fourth decimal place are correct. **DO NOT WRITE YOUR ANSWERS AS FRACTIONS.** ½ should be written as 0.5.

Set your calculator to radian mode.

1.		14.	
2.		15.	
3.		16.	
4.		17.	
5.		18.	
6.		19.	
7.		20.	
8.		21.	
9.		22.	
10.		23.	
11.		24.	
12.		25.	
13.		26.	

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1. $\frac{5137}{2341} =$

2. Give the slope of the line that passes through the y-intercept of the graph of $f(x) = 1/(3x+7)^4$ and the largest x-intercept of $g(x) = 13x^3 - 17x + 56$.

3. Give the smallest integer larger than

$$\left(\frac{(1376 - 24^3)}{(17^2 + 11)^3} - 17\right)^2 + 2$$

4. Give the x-intercept of the line of best least squares fit for the data $(-1.3, 2.7)$, $(1.4, 5.5)$ and $(3.9, 8.2)$.

5. Give the products of the smallest and largest solutions to $13x^6 - 14x^4 + 13x = 17$.

6. $h(x) = \frac{\frac{\frac{1}{x}}{x+1}}{x+3}$. Define $g(x) = h\left(h\left(h(h(x)+2)\right)\right)$. Find $g(1)$.

7. Determine the number of points of intersection of the graphs of $y = 2x - 1$ and

$$f(x) = 5.82 \cdot 10^{-7}x^8 - 2.24 \cdot 10^{-5}x^7 - 5.15 \cdot 10^{-4}x^6 + 1.99 \cdot 10^{-3}x^5 - 0.14x^3 + 4.62x^2 + 0.506x - 29.89$$

8. Solve the system $\begin{pmatrix} 72x - 81y = 216 \\ 137x + 83y = 111 \end{pmatrix}$, and then give the value of $27x - 36y$.

9. The graph of $y = ax^2 + bx + c$ passes through the points $(-1,2)$, $(1,3)$ and $(4,-9)$. Give the value of a.

10. If a and b are positive integers, then $mod_a b$ is equal to the remainder that occurs when b is divided by a. For example, $mod_{19} 81 = 5$. Give $mod_{277} 72341$.

11. If p is a prime number and $b \in \{0,1,\dots,p-1\}$, then we define $[b]_p$ to be the unique value in $\{0,1,\dots,p-1\}$ so that $mod_p\left(b \cdot [b]_p\right) = 1$. And if $b, c \in \{0,1,\dots,p-1\}$, we define $(c,b)_p = mod_p\left(c * [b]_p\right)$. Give $[216]_{277}$.

12. Referring to problems 10 and 11. Give $(3, 111)_{277}$.

13. Referring to problems 10 and 11. Give $mod_{277}(14 \cdot (13, 176)_{277} + 1321)$.

14. Evaluate: $1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \frac{1}{11} + \cdots + \frac{1}{1001} =$

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15. A point (a, b) in the first quadrant of the xy-plane is called an *integer point* if a and b are positive integers. Give the number of integer points in the first quadrant that lie below the graph of $y = 10\sin(x)$ for $0 \le x \le 500$.

16. If a, c and d are real numbers, then the matrix $\begin{pmatrix} a & c \\ c & d \end{pmatrix}$ is *negative definite* if and only if $a < 0$ and $ad - c^2 > 0$. What is the probability that a randomly chosen matrix of the form $\begin{pmatrix} a & c \\ c & d \end{pmatrix}$, with
$$a, c, d \in \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$$
is negative definite?

17. The function $g(x) = 4x^3 + 12x + \cos(x)$ is invertible. Find $g^{-1}(31)$.

18. A quadrilateral has its vertices listed in clockwise order as
$$(-6.94,\ -1.2), (-5.16, 3.84), (3.4, 2.18) \text{ and } (2.46,\ -6.72).$$
Give the area of this quadrilateral.

19. Give the circumference of the quadrilateral in the problem above.

20. A sequence $a_1, a_2, a_3, \ldots$ is created by setting $a_1 = 0$. Then, for each $n = 1,2,3, \ldots$, if $a_n = k$, and the only solution to $a_j = k$ for $j \in \{1, \ldots, n\}$ is $j = n$, then $a_{n+1} = 0$. Otherwise, if $j \in \{1, \ldots, n-1\}$ is the largest value for which $a_j = k$, then $a_{n+1} = n - j$. The terms of the sequence are
0, 0, 1, 0, 2, 0, 2, 2, 1, 6, 0, 5, 0, 2, 6, 5, 4, 0, 5, 3, 0, 3, …
We name these $a_1,\ a_2,\ a_3,\ a_4,\ a_5, \ldots$ Give the value of a_{147}.

21. Jenny invests \$10,000 in a mutual fund, and the annual returns are 3.1%, 4.2%, 10.3%, -4.1%, 7.2%, 6.3%, 9.1%, -4.2%, 8.4% and 6.7% during the first 10 years of the investment. How much money is in the account after 10 years?

22. One approximation of π is given by
$$p_n = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{(-1)^{n+1}}{2n-1}\right)$$
where n is a positive integer larger than 4. A simpler approximation of π is given by $\frac{22}{7}$. Give the smallest value of n so that $|p_n - \pi| < \left|\frac{22}{7} - \pi\right|$.

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23. Pascal's triangle has the form

$$\begin{array}{cccccccc} 1 \\ 1 & 1 \\ 1 & 2 & 1 \\ 1 & 3 & 3 & 1 \\ 1 & 4 & 6 & 4 & 1 \\ 1 & 5 & 10 & 10 & 5 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{array}$$

We assign numbers to the locations in Pascal's triangle as shown below,

$$\begin{array}{cccccccc} 1 \\ 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 & 21 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{array}$$

and we let L be the function that assigns the location numbers to the entries in Pascal's triangle. This gives $L(3) = 1$, $L(13) = 6$, etc. Give the value of $L(337)$.

24. Give the number of values in the set $S = \{1, 2, 3, 4, 5, \ldots, 10^4\}$ that are not divisible by any of 3, 5, 7, 11 or 13.

25. Give the average of the answers to problems 1 through 25.

26. **Tie Breaker:** The hat function is defined by

$$h(x) = \begin{cases} 0, & \text{if } |x| > 1 \\ 1 - |x|, & \text{if } |x| \le 1 \end{cases}$$

Create the function

$$f(x) = \sum_{i=1}^{82} [i]_{83} h(x - i)$$

where $[i]_{83}$ is defined in problem 11. Give the length of the graph of $y = f(x)$ for $-5 \le x \le 90$.