UNIVERSITY OF HOUSTON
HIGH SCHOOL MATHEMATICS CONTEST

Spring 2019 Calculus Test

NAME: ____

SCHOOL: ____

1. $\displaystyle\lim_{x\to\infty} \frac{5+2x-x^3}{\sqrt{4x^5+2x^3+x+1}} =$

   (a) $1/2$
   (b) 0
   (c) -1
   (d) $-1/2$
   (e) The limit does not exist.

2. $\displaystyle\lim_{x\to 0} 3\sin\left[\frac{3\sin(2\pi x)}{8x}\right] =$

   (a) $-3\sqrt{2}/2$
   (b) $-3\sqrt{3}/2$
   (c) $3\sqrt{2}/2$
   (d) $3/2$
   (e) The limit does not exist.

3. $\displaystyle\lim_{h\to 0} \frac{\cos(\pi/6+4h)-\cos(\pi/6)}{h} =$

   (a) -2
   (b) $2\sqrt{3}$
   (c) 2
   (d) 0
   (e) $-2\sqrt{3}$

4. Set

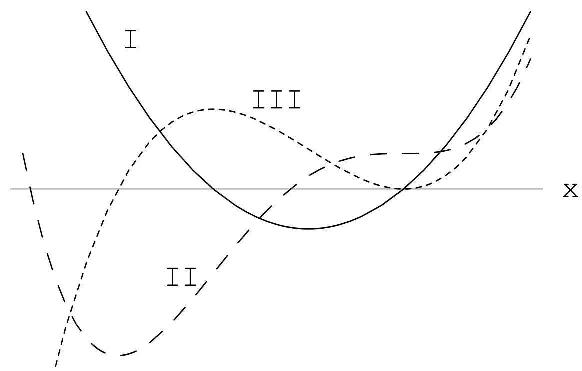
$$g(x) = \begin{cases} x^3 - 2x & x < 2, \\ ax^2 + bx & x \geq 2. \end{cases}$$

If g is everywhere differentiable, then $a + b =$

(a) 4
(b) -6
(c) 2
(d) 10
(e) -2

5. Three graphs labeled I, II and III are shown in the figure. One is the graph of f, one is the graph of f' and one is the graph of f''. Which of the following correctly identifies each of the three graphs?

	f	f'	f''
(a)	I	II	III
(b)	I	III	II
(c)	II	I	III
(d)	II	III	I
(e)	III	II	I



6. Let f and g be differentiable functions which satisfy the following conditions:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	3	2	4
2	-1	-2	-1	0

If $h(x) = f(g(x))$, then $h'(0) =$

(a) -4
(b) -8
(c) 0
(d) 12
(e) 8

7. If $f(x) = 4^x \ln(3e^x)$, then $f'(0) =$

(a) $4\ln(2) + 4$
(b) $\ln(4)\ln(3) + 1$
(c) $\ln(4)\ln(3) + 4$
(d) $\ln(12) + 1$
(e) $4\ln(7) + 1$

8. When the linear approximation of $f(x) = \sqrt{4 + \ln x}$ near $x = 1$ is used, an estimate of $f(1.08)$ is:

(a) 1.98
(b) 2.01
(c) 2.02
(d) 2.04
(e) 2.06

9. Suppose that f is continuous on $[0, 4]$ and differentiable on $(0, 4)$. Suppose also that $f(0) = 5$ and $f(4) = -3$. Which of the following statements is not necessarily true?

I. There exists a number $c \in (0, 4)$ such that $f'(c) < -1$
II. There exists a number $c \in (0, 4)$ such that $f(c) = \pi$.
III. There exists a number $c \in (0, 4)$ such that $f'(c) = 2$.
IV. If $c \in (0, 4)$, and $f'(c) = 0$, then $f(c)$ is either a maximum of a minimum of f on $[-1, 3]$

(a) II, IV
(b) I, II
(c) II, III, IV
(d) III, IV
(e) III only

10. The position of a particle moving along a horizontal line is given by

$$s(t) = \int_0^t (u^2 - 6u + 5)\, du, \quad 0 \le t \le 10,$$

where t represents time. The interval(s) on which the speed of the particle is increasing is(are):

(a) $(1, 3)$ and $(5, 10]$
(b) $[0, 1)$ and $(3, 5)$
(c) $(3, 10]$
(d) $[0, 1)$ and $(3, 10]$
(e) $(1, 5)$

11. A point (x, y) is moving along a curve $y = f(x)$. At the instant when the slope of the curve is $-2/3$, the x-coordinate of the point is decreasing at the rate of 5 units per second. The rate of change, in units per second, of the y-coordinate is

(a) $15/2$
(b) $-10/3$
(c) $3/10$
(d) $-2/15$
(e) $10/3$

12. A function f is differentiable and decreasing on $(-\infty, \infty)$. If $g(x) = f(x^3 - 3x^2 - 9x)$, then g has a local maximum at:

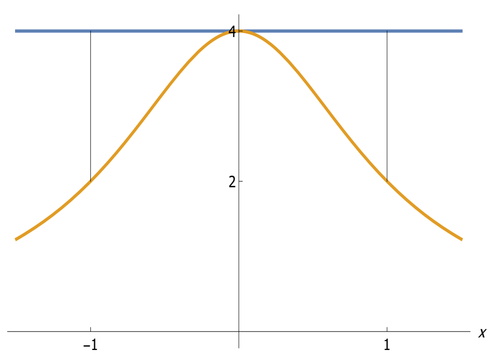
(a) $x = 3$
(b) $x = 1$
(c) $x = 0$
(d) $x = -1$
(e) There is no local maximum.

13. An equation for the normal line to the curve $2x^3 + 2y^2 = 5xy$ at the point $(1, 2)$ is

(a) $3x - 4y = 2$
(b) $3x + 4y = -5$
(c) $3x + 4y = 11$
(d) $4x - 3y = -2$
(e) $4x + 3y = 10$

14. In square units, the area of the region bounded above by the the line $y = 4$, below by the graph of $f(x) = \dfrac{4}{1 + x^2}$, and on the sides by the lines $x = \pm 1$ (see the figure), is:

(a) $4 - \pi/4$
(b) $8 - \pi/2$
(c) $8 - \pi$
(d) $8 - 2\pi$
(e) $2\pi - 4$



15. The base of a solid is the region in the first quadrant of the xy-plane bounded by $x^2 = 4y$, the y-axis and the line $y = 2$. Each plane section of the solid perpendicular to the y-axis is a semi-circle. The volume of the solid in cubic units is:

(a) $\pi/2$
(b) 2π
(c) π^2
(d) 4π
(e) π

16. The region bounded by the graph of $f(x) = \sqrt{x-1}$, the vertical line $x = 10$, and the x-axis is revolved about the line $y = 3$. The volume of the generated solid, in cubic units, is:

(a) $\dfrac{99\pi}{2}$
(b) $\dfrac{189\pi}{2}$
(c) $\dfrac{135\pi}{2}$
(d) $\dfrac{119\pi}{2}$
(e) $\dfrac{137\pi}{2}$

17. If length is measured in centimeters, then the length of the graph of $f(x) = \ln(\sec x)$, where $0 \le x \le \pi/3$, is:

(a) $3 + \sqrt{2}$ cm
(b) $\ln\left(2 + \sqrt{3}\right)$ cm
(c) $\ln\left(\sqrt{3}\right)$ cm
(d) $2 + \sqrt{3}$ cm
(e) $\ln\left(\dfrac{1 + \sqrt{3}}{2}\right)$ cm

18. A curve in the plane has the property that the normal line to the curve at each point $P(x, y)$ always passes through the point $(0, 2)$. Find an equation for the curve given that it passes through the point $(3, 1)$.

(a) $x^2 + (y-2)^2 = 10$
(b) $(x-2)^2 + 2y^2 = 3$
(c) $y - 2 = x^2 - 10$
(d) $(x-2)^2 + y^2 = 2$
(e) $\dfrac{x^2}{3} + 2(y-2)^2 = 5$

19. Let f be a continuous function on $(-\infty, \infty)$. If $F(x) = \int_2^x x^2 f(t)\, dt$, then $F''(2) =$

(a) $4f(2) + 4f'(2)$
(b) $8f(2) + 4f'(2)$
(c) $6f(2) + 4f'(2) + 2f''(2)$
(d) $8f'(2) + 4f''(2)$
(e) $4f(2) + 8f'(2)$

20. The function $F(x) = 2x + \int_{x^2}^4 \sqrt{4+3t}\, dt$ has an inverse. $\left(F^{-1}\right)'(4) =$

(a) $1/6$
(b) $-1/12$
(c) $1/18$
(d) $-1/14$
(e) $-1/18$

21. Find k if the average value of $f(x) = x^3 + 1$ on $[0, k]$ is 17.

(a) 6
(b) $\sqrt[4]{68}$
(c) 4
(d) 3
(e) $\sqrt{48}$

22. $\displaystyle\int_1^2 \frac{x^2}{\sqrt{4-x^2}}\, dx$ is equivalent to:

(a) $4\displaystyle\int_1^2 \sin^2\theta\, d\theta$
(b) $2\displaystyle\int_0^{\pi/2} \sin\theta \tan\theta\, d\theta$
(c) $2\displaystyle\int_{\pi/6}^{\pi/2} \frac{\sin^2\theta}{\cos\theta}\, d\theta$
(d) $4\displaystyle\int_0^{\pi/2} \sin^2\theta\, d\theta$
(e) $4\displaystyle\int_{\pi/6}^{\pi/2} \sin^2\theta\, d\theta$

23. If $\int_0^8 e^x \, dx = A$, then $\int_0^2 x^2 e^{x^3} \, dx =$

(a) $\frac{1}{3}A$
(b) A^3
(c) $\frac{1}{3}A^3$
(d) $3A$
(e) A

24. Find the number(s) a such that $\lim_{x \to 0} \frac{e^{a^2x^2} - \cos(4x)}{x^2} = 12.$

(a) $a = \pm\sqrt{12}$
(b) $a = 2, \; 4$
(c) $a = \pm\sqrt{20}$
(d) $a = -4, \; 4$
(e) $a = -2, \; 2$

25. The general solution of the differential equation $\frac{dy}{dx} = \frac{1 - 2x}{y}$ is a family of:

(a) straight lines
(b) circles
(c) ellipses
(d) parabolas
(e) hyperbolas

26. The rate at which a certain bacteria population grows is proportional to number of bacteria present. Initially there were 1,000 bacteria present and the population doubled in 6 hours. Approximately how many hours will it take for the population to reach 10,000?

(a) 17
(b) 31
(c) 14
(d) 20
(e) 24

27. $\lim_{x \to 0} (1 + 2x)^{1/x} =$

(a) 0
(b) 2
(c) e
(d) e^2
(e) The limit does not exist.

28. If a block of ice melts at the rate of $\dfrac{72}{2t+3}$ cm^3/min, then the closest approximation to the amount of ice which melts during the first three minutes is:

(a) $40\ \text{cm}^3$
(b) $44\ \text{cm}^3$
(c) $36\ \text{cm}^3$
(d) $32\ \text{cm}^3$
(e) $48\ \text{cm}^3$

29. Set $f(x) = \begin{cases} 2x+1, & 1 \le x < 3 \\ 4, & 3 \le x \le 5 \end{cases}$ If $F(x) = \int_1^x f(t)\,dt$, then $F(4) =$

(a) 16
(b) 12
(c) 26
(d) 14
(e) 17

30. $\displaystyle \lim_{n\to\infty} \sum_{k=0}^{n} \frac{1}{n\left(1+\frac{k}{n}\right)} =$

(a) 2
(b) $\ln 2$
(c) $2\ln 2$
(d) $\ln 2 - 1$
(e) 1