

Name: _____ School: _____

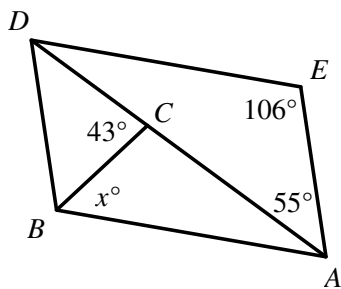
University of Houston Mathematics Contest Geometry Exam – Spring 2019

Answer the following. Note that diagrams may not be drawn to scale.

1. Given kite $ABCD$, where \overline{AC} is the perpendicular bisector of \overline{BD} . Find CD , given that $AB = 5$ and the perimeter of the kite is 24.

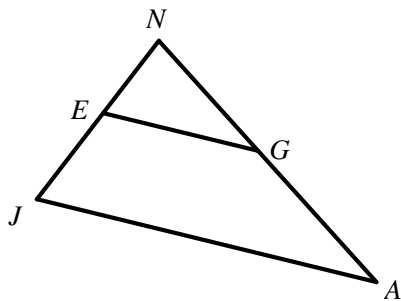
A. 19 B. 5 C. 7 D. 17 E. 9.5 F. None of these

2. Parallelogram $BDEA$ is shown below. Find the value of x .



- A. 55
B. 24
C. 43
D. 19
E. 14
F. None of these

3. In the diagram below, $\overline{EG} \parallel \overline{JA}$, $EN = x + 9$, $JE = 28$, $JA = 9x + 9$, $AG = 30$, and $NG = 2x + 5$. Find EG .

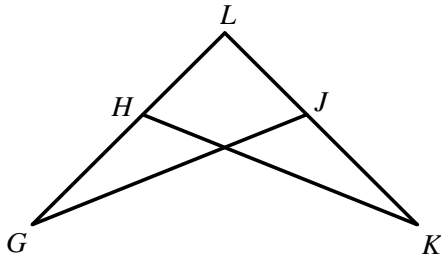


- A. 45
B. 27
C. 42
D. 54
E. 12
F. None of these

4. A regular convex polygon has exactly six lines of symmetry. Find the sum of the interior angles of the polygon.

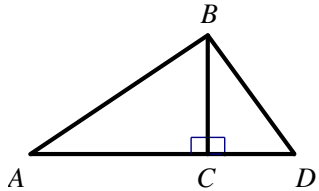
A. 1080° B. 180° C. 2160° D. 1800° E. 720° F. None of these

5. In the diagram below, $\overline{HL} \cong \overline{JL}$ and $\overline{LK} \cong \overline{LG}$. Which theorem or postulate can be used to prove that $\triangle GLJ \cong \triangle KLH$?



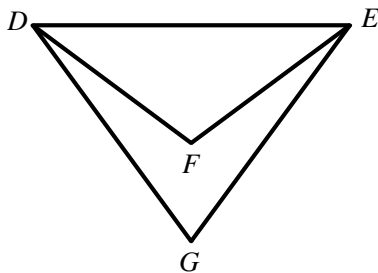
- A. Side-Side-Side
- B. Angle-Side-Angle
- C. Side-Angle-Side
- D. Hypotenuse-Leg
- E. Angle-Angle-Side
- F. The triangles are not necessarily congruent.

6. In the diagram below, $BD = 16$, $m\angle D = 45^\circ$, and $m\angle A = 60^\circ$. Find AB .



- A. $8\sqrt{6}$
- B. $\frac{16\sqrt{6}}{3}$
- C. $16\sqrt{2}$
- D. $\frac{16\sqrt{3}}{3}$
- E. $8\sqrt{3}$
- F. None of these

7. $\triangle DEF$ and $\triangle DEG$ are isosceles triangles with base \overline{DE} . If $\angle EDG$ measures a° more than $\angle DEF$, and $m\angle G = b^\circ$, find $m\angle DFE$.

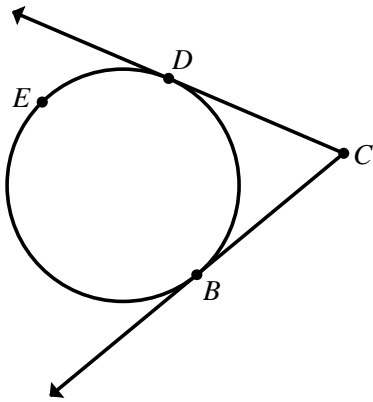


- A. $(180 - 2a + b)^\circ$
- B. $(360 - 2a - b)^\circ$
- C. $(2a - b)^\circ$
- D. $(2a + b)^\circ$
- E. $(2a)^\circ$
- F. None of these

8. How many distinct lines are determined by seven points, provided that exactly three of them are collinear?

- A. 19
- B. 18
- C. 21
- D. 15
- E. 20
- F. None of these

9. Find $m\widehat{DB}$, given that $m\angle C = 48^\circ$.



- A. 108°
- B. 96°
- C. 132°
- D. 156°
- E. 48°

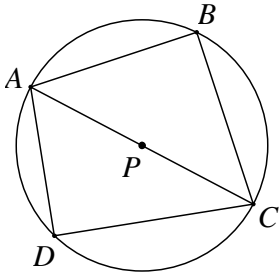
10. Given the following conditional statement:

If a pentagon is equiangular, then it is equilateral.

Determine the truth value of the converse, inverse, and contrapositive, respectively.

- | | | |
|-----------------------|-----------------------|------------------------|
| A. True, True, True | B. True, True, False | C. False, False, False |
| D. True, False, False | E. False, False, True | F. None of these |

11. Quadrilateral $ABCD$ is inscribed in circle P as shown below. If $AB = x - 4$, $AD = x + 4$, $BC = 3x - 9$, and $DC = 2x - 2$, find the perimeter of $ABCD$.



- A. $\frac{167 + 7\sqrt{533}}{2}$
- B. 60
- C. 56
- D. 66
- E. 91
- F. None of these

12. The area of a trapezoid is 120 cm^2 , and its height is 8 cm. Find the length of the median of the trapezoid, in centimeters.

- A. 7.5
- B. 30
- C. 15
- D. 8
- E. 60
- F. None of these

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13. In spherical geometry, a line is defined to be a great circle on the surface of a sphere. Suppose that a sphere has a diameter of 18 cm. Given the definition above, what is the minimum number of lines needed to divide the sphere into sections of the same shape and size, where each section has an area of $27\pi \text{ cm}^2$?
- A. 12 B. 4 C. 6 D. 25 E. 24 F. No solution
14. A regular pentagon is circumscribed about a circle of radius 4. Find the area of the regular pentagon.
- A. $80 \sin(36^\circ)$ B. $80 \tan(36^\circ)$ C. $40 \sin(36^\circ)$
D. $\frac{20}{\tan(36^\circ)}$ E. $40 \tan(36^\circ)$ F. $\frac{40}{\tan(36^\circ)}$
15. Angela and Bob are in line at a cafeteria, and there are two pizzas of equal thickness, denoted Pizza A and Pizza B. Angela buys a slice of pizza from Pizza A that has a central angle of 80° . Bob buys a slice of pizza from Pizza B that has a central angle of 40° . If the diameter of Pizza B is twice the diameter of Pizza A, the volume of Angela's slice is _____ the volume of Bob's slice.
- A. twice B. half C. the same as
D. four times E. one-fourth of F. None of these
16. Find the x -intercept of the line that is perpendicular to $y = \frac{3}{5}x - 1$ and passes through the point $(6, -8)$.
- A. $\frac{6}{5}$ B. $\frac{54}{5}$ C. 2 D. $-\frac{22}{3}$ E. $\frac{5}{3}$ F. None of these
17. The hour hand of a clock is 15 cm long. On any given day, how far (in centimeters) does the tip of the hour hand travel between 4:47 p.m. and 8:11 p.m.?
- A. $\frac{23\pi}{2}$ B. 9π C. $\frac{17\pi}{2}$ D. $\frac{255\pi}{4}$ E. 11π F. None of these

18. Five diagrams are given below.

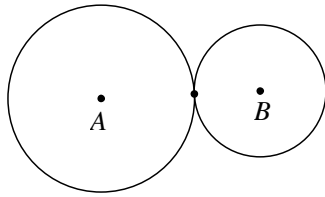


Figure 1

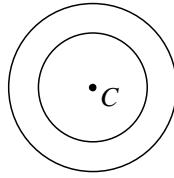


Figure 2

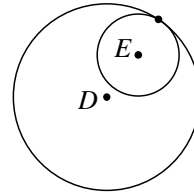


Figure 3

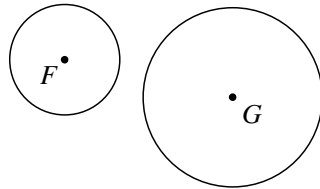


Figure 4

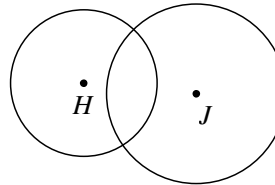


Figure 5

Based on the above diagrams, which of the following statements are true?

Create an answer using the numbers associated with the true statements. For example, if only 1, 2, and 5 are true, then the answer is 125; if only 3 and 5 are true, then the answer is 35, etc.

1. Circles H and J have no common external tangents.
2. Circles A and B are externally tangent.
3. Circles F and G have exactly two common internal tangents.
4. The circles in Figure 2 are concentric circles.
5. Circles A and B have exactly two common tangents.
6. Circles D and E are externally tangent.
7. Circles F and G have exactly four common external tangents.

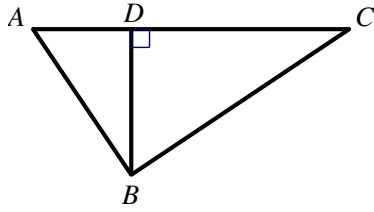
- A. 124567 B. 245 C. 134 D. 234
 E. 236 F. 23456 G. None of these

19. $\triangle ABC$ forms the base of a right triangular prism. If $AC = 3$ cm, $BC = 4$ cm, and the height of the prism is 10 cm, which of the following numbers could represent the lateral area of the prism, in square centimeters?

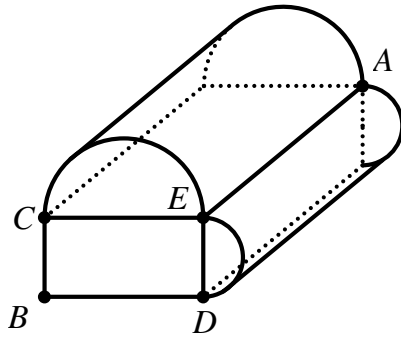
- I. 60 II. 132 III. 120
 IV. 150 V. $70 + 10\sqrt{7}$ VI. 80

- A. I only E. II, III, and V only
 B. II only F. II, III, V, and VI only
 C. III only G. II, III, IV, V, and VI only
 D. III and V only H. I, II, III, IV, V, and VI

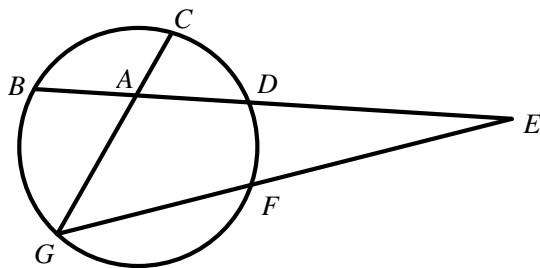
20. Find the length of the longer leg of right triangle ABC , given that $BD = 6$ and $AC = 13$.



- A. $9\sqrt{13}$ B. 9 C. $2\sqrt{13}$ D. 58.5 E. $3\sqrt{13}$ F. None of these
21. In the three dimensional solid below, all polygon sides meet at right angles, and all curves are semicircles. If $BD = 8$, $CB = 6$, and $BA = 15$, find the volume of the solid.

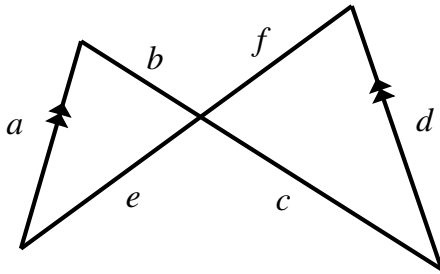


- A. $140\sqrt{5} + 35\pi\sqrt{5}$
 B. $240\sqrt{5} + \frac{125\pi\sqrt{5}}{2}$
 C. $720 + \frac{375\pi}{2}$
 D. $240\sqrt{5} + 35\pi\sqrt{5}$
 E. $420 + 125\pi\sqrt{5}$
 F. None of these
22. In the circle below, $m\angle CAD = 68^\circ$, $m\widehat{CD} = 40^\circ$, and $m\widehat{DF} = 22^\circ$. Find $m\angle E$.



- A. 48°
 B. 37°
 C. 9°
 D. 23°
 E. 59°
 F. None of these

23. In the diagram below, $a, b, c, d, e,$ and f represent lengths of segments as shown in the diagram below. Note the two parallel segments with measures a and d .



Which of the following statements are true?

Create an answer using the numbers associated with the true statements. For example, if only 1, 2, and 5 are true, then the answer is 125.

1. $\frac{a-d}{d} = \frac{b-c}{c}$

2. $\frac{a+f}{d+f} = \frac{b+f}{c+f}$

3. $\frac{d+c}{a+b} = \frac{f}{e}$

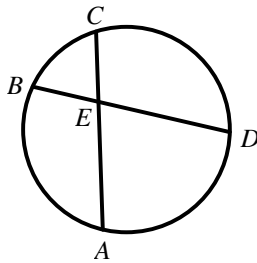
4. $\frac{a+b+e}{d+f+c} = \frac{b}{f}$

5. $\frac{e+c}{c} = \frac{b+f}{f}$

6. $\frac{a+b+e}{d+f+c} = \frac{a}{d}$

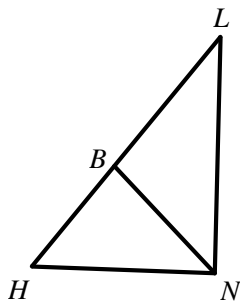
- A. 126 B. 145 C. 456 D. 1236 E. 136 F. None of these

24. Chords \overline{AC} and \overline{BD} intersect at point E , $AC = 12$, $BE = 6$, and $ED = 4$. If $AE \geq EC$, find the value of $AE - EC$.



- A. 7
 B. $4\sqrt{3}$
 C. 0
 D. 10
 E. $8\sqrt{3}$
 F. No solution

25. In the diagram below, $BH = 2x - 4$, $LN = 15$, $HN = 7x - 8$, $BL = x - 1$, and \overline{NB} is an angle bisector of $\triangle LHN$. Find HL .

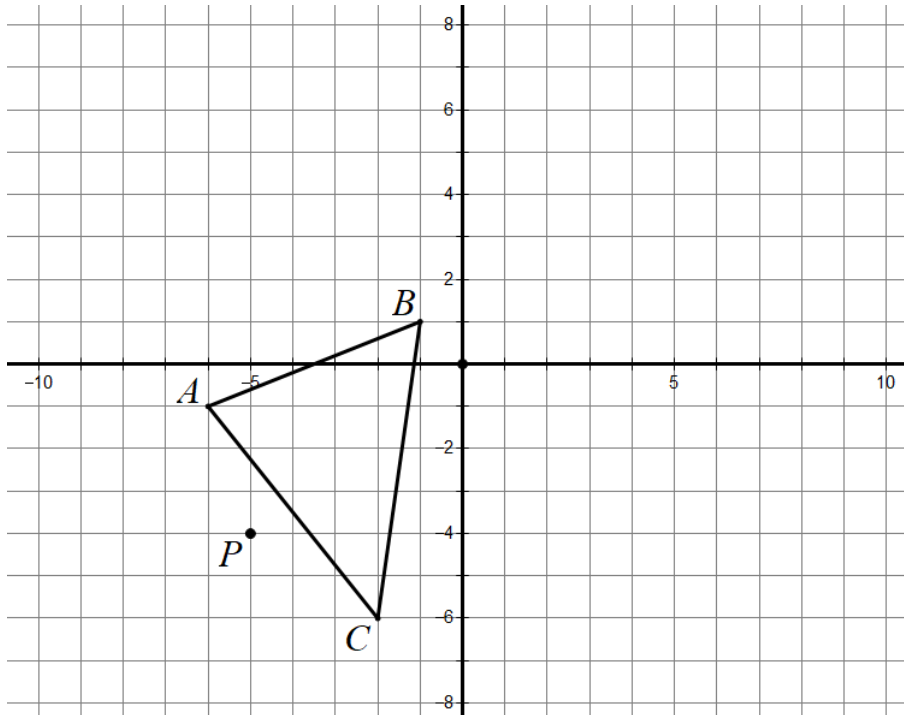


- A. 25 B. 10 C. 22 D. 7 E. $\frac{16}{7}$ F. None of these

26. Regular hexagon ABCDEF with side length 6 is rotated about diagonal AD, tracing out a three-dimensional solid. Find the surface area of the solid formed.

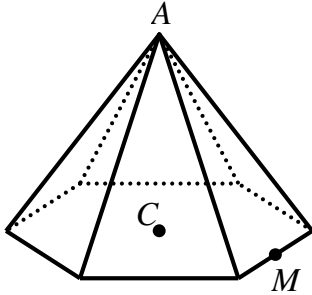
- A. 108π
- B. 216π
- C. $72\pi\sqrt{3}$
- D. 126π
- E. $54\pi\sqrt{3}$
- F. None of these

27. $\triangle ABC$ is shown below with $A(-6, -1)$, $B(-1, 1)$, and $C(-2, -6)$. Suppose that $\triangle ABC$ is dilated about point $P(-5, -4)$ with scale factor 200% to form the image $A'B'C'$. Find the equation of the line joining the midpoints of $\overline{A'B'}$ and $\overline{B'C'}$.



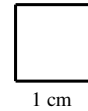
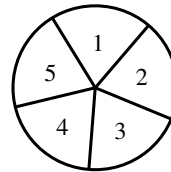
- A. $5x + 4y = 6$
- B. $10x + 8y = -105$
- C. $2x + 2y = 3$
- D. $10x + 8y = 59$
- E. $5x + 4y = -35$
- F. None of these

28. A right regular hexagonal pyramid is shown below. The hexagonal base has a side length of 10 cm, and the volume of the pyramid is $750\sqrt{2} \text{ cm}^3$. If A represents the apex of the pyramid, C represents the center of the base, and M represents the midpoint of one of the sides of the hexagon, find the sine of $\angle AMC$.



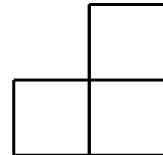
- A. $\frac{\sqrt{15}}{5}$ B. $\frac{\sqrt{6}}{3}$ C. $\sqrt{2}$ D. $\frac{\sqrt{3}}{3}$ E. $\frac{\sqrt{22}}{11}$ F. None of these

29. Lori is playing a game involving a spinner and tiles. The spinner is divided into five equal portions, all of which are equally likely. Each thin square tile is 1 cm on each edge.



If Lori spins a “1”, she receives one tile. If she spins a “3”, she receives 3 tiles, etc. The tiles are magnetic on their edges, and upon receiving the tiles, Lori needs to assemble them so that each tile is sharing at least one full edge with another tile. (The tiles cannot be stacked on top of each other.)

If Lori spins a “3” on her first spin, for example, she might assemble her tiles like one of the designs shown at the right:



On every ensuing spin, Lori receives the indicated number of tiles, and connects those pieces to her existing tile design. If Lori starts the game over (with zero tiles in front of her) and spins the spinner twice, what is the probability that she will have enough tiles to make a design with a perimeter of exactly 12 cm?

- A. $\frac{9}{25}$ B. $\frac{18}{25}$ C. $\frac{2}{25}$ D. $\frac{2}{5}$ E. $\frac{14}{25}$ F. None of these

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Questions 30-32 are write-in answers rather than multiple choice. Write the answer to each question on the answer sheet in the space provided. All irrational answers (containing radicals or π) should be left as exact answers rather than decimal approximations. All radicals and fractions should be written in simplest form, with no radicals in the denominator. Do not write any units on the answer sheet.

30. The distance between $\left(\frac{1}{4}, -\frac{5}{6}\right)$ and $\left(\frac{3}{4}, n\right)$ is $\frac{5}{6}$. Find all possible values of n .

31. Suppose that the following three circles are drawn on the coordinate plane:

$$x^2 + y^2 + 6x - 10y + 25 = 0$$

$$x^2 + y^2 - 2x - 6y - 54 = 0$$

$$x^2 + y^2 - 8x + 12 = 0$$

If a point is randomly chosen within the largest circle, what is the probability that the point lies within one of the two smaller circles?

Continued on the next page...

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32. The base of a hemisphere is placed on the base of a square pyramid so that the hemisphere is tangent to all four lateral edges of the pyramid. If the pyramid has a base edge of 3 cm and a height of 6 cm, find the volume of the hemisphere.

END OF EXAM ☺