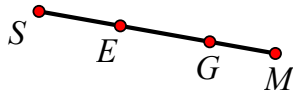


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University of Houston Mathematics Contest Geometry Exam – Spring 2020

Answer the following. Note that diagrams may not be drawn to scale.

1. How many distinct segments can be named using the diagram below?



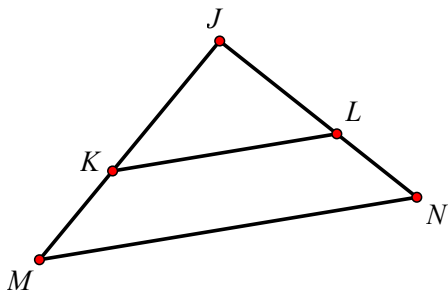
- A. 3 B. 4 C. 6 D. 8 E. 12 F. None of these

2. Given the following conditional statement,

If M is the midpoint of \overline{AB} , then $\overline{AM} \cong \overline{MB}$.

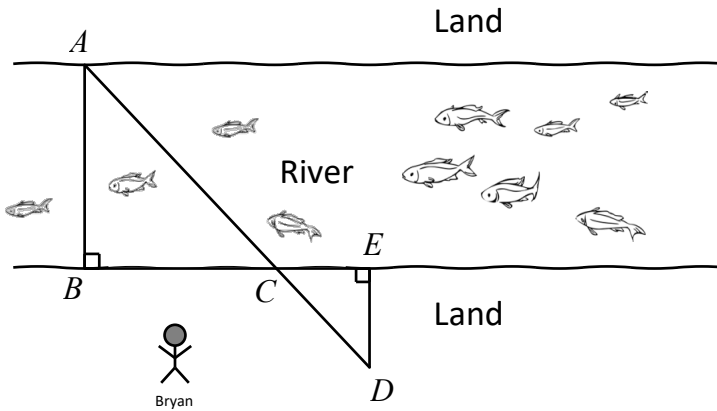
Find the truth values of the converse, contrapositive, and inverse, respectively.

- A. False, False, False B. False, True, False C. True, True, True
D. True, False, False E. True, True, False F. None of these
3. In the diagram below, $\overline{KL} \parallel \overline{MN}$, $KM = x - 2$, $JL = x - 1$, $LN = 12$, and $JK = 6$. Find JN .

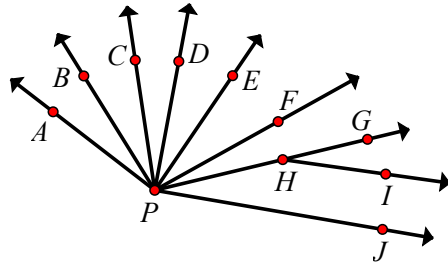


- A. 9 B. 7 C. 10 D. 21 E. 18 F. None of these
4. The measure of a cylinder's height is $\frac{2}{3}$ of its diameter.. Find the lateral area of the cylinder, given that its volume is 288π cubic units.
- A. 168π B. 1152π C. 132π D. 96π E. 384π F. None of these

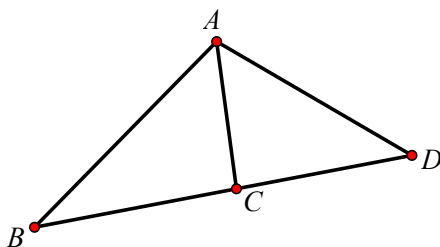
5. Bryan wants to estimate the distance across the river while staying on land. He takes measurements and finds that $BC = 1200$ ft, $CE = 80$ ft, and $DE = 120$ ft. Based on Bryan's measurements, find AB , the distance across the river, in feet.



- A. 1600 B. 1800 C. 800 D. 1240 E. 1160 F. None of these
6. How many distinct angles in the following diagram are adjacent to $\angle EPG$? (Assume that the diagram is drawn to scale, and do not count any angles with measures greater than 180 degrees.)

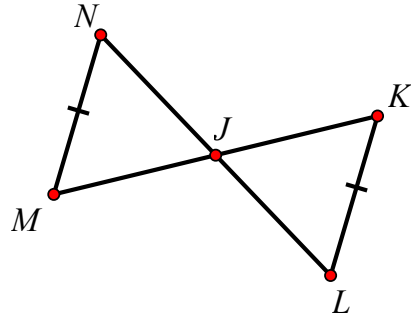


- A. 2
B. 3
C. 6
D. 7
E. 8
F. None of these
7. In the diagram below, $AB = 10$, $BC = x - 3$, $CD = 2x - 9$, $AD = 14$, and \overline{AC} is an angle bisector. Find BD .



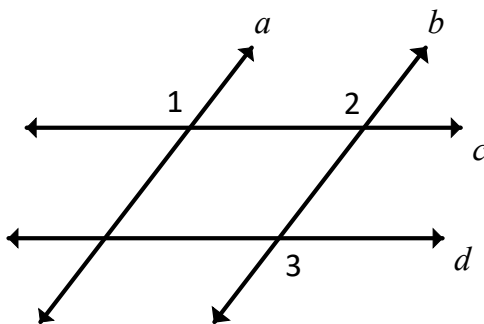
- A. 12
B. 22
C. 54
D. 8
E. $3 + 3\sqrt{33}$
F. None of these

8. Given the information in the diagram below, which of the following case(s) provide enough additional information to prove that $\triangle KJL \cong \triangle MJN$? (Treat each case as separate from the others, and decide if the information for that case is sufficient along with the diagram to show that $\triangle KJL \cong \triangle MJN$.)



- Case 1: $\overline{JK} \cong \overline{NJ}$
 Case 2: $\angle N \cong \angle L$
 Case 3: $\overline{NJ} \cong \overline{JL}$
 Case 4: $\overline{NJ} \perp \overline{JK}$ and $\overline{MJ} \cong \overline{JK}$
 Case 5: $\angle L \cong \angle M$
 Case 6: $\overline{MJ} \perp \overline{JL}$ and $\overline{KJ} \cong \overline{JN}$

- A. 123456 B. 156 C. 24
 D. 234 E. 12456 F. None of these
9. In spherical geometry, a line is defined to be a great circle on the surface of a sphere. If two lines on a sphere are perpendicular, how many right angles are formed?
- A. 1 B. 4 C. 16 D. 2 E. 8 F. None of these
10. In the diagram below, $m\angle 1 = (3x + 10)^\circ$, $m\angle 2 = (2x + 60)^\circ$, and $m\angle 3 = (4x - 30)^\circ$. What value of x is needed to prove that $c \parallel d$?



- A. 45
 B. 22
 C. 50
 D. 40
 E. $\frac{200}{7}$
 F. None of these

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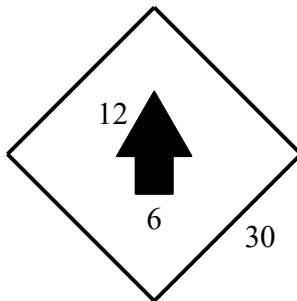
11. The tires on Jay's bicycle each have a radius of 15 inches. If Jay bikes for four minutes while his tires are rotating at a constant speed of 200 revolutions per minute, how many feet has Jay traveled?

A. $1,000\pi$ B. $2,000\pi$ C. $288,000\pi$
D. $24,000\pi$ E. 500π F. None of these

12. An interior angle of a regular polygon is four times as large as an exterior angle of that same polygon. How many sides does the polygon have?

A. 10 B. 6 C. 12 D. 5 E. 9 F. None of these

13. A square street sign contains a picture of a painted arrow. The arrow is formed from an equilateral triangle on top of a square. If a small bug lands in a random location on the sign, find the probability that the bug lands within the arrow. (Assume that the bug is equally likely to land at any point on the sign.)



A. $\frac{3\sqrt{3}+3}{10}$ B. $\frac{2\sqrt{3}+1}{25}$ C. $\frac{2}{25}$
D. $\frac{\sqrt{3}+1}{25}$ E. $\frac{3\sqrt{3}+3}{5}$ F. None of these

14. Tim has a box of straws that are each 9 inches long. He takes out three straws and trims some or all of them so that they form lengths of x , $x+1$, and $x+4$ inches. He then places the three trimmed pieces together to form a triangle. How many such triangles can Tim create, if the perimeter of each triangle must be a whole number? (Assume that no two completed triangles are congruent, and assume that Tim does not run out of straws.)

A. 2 B. 4 C. 5 D. 6 E. 15 F. None of these

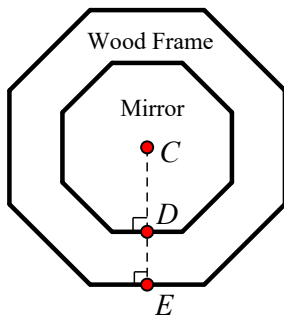
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15. Given the following points and their coordinates: $A(-5, 3)$, $B(2, 7)$, and $C(3, -1)$
Which of the following statements are true?

- I. A lies on the perpendicular bisector of \overline{BC} .
- II. B lies on the perpendicular bisector of \overline{AC} .
- III. C lies on the perpendicular bisector of \overline{AB} .

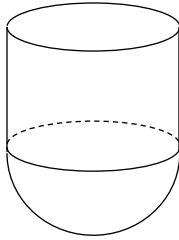
- A. I only B. II only C. III only D. I and II only
E. I and III only F. II and III only G. I, II, and III H. None of these

16. A regular octagonal mirror is surrounded by a wooden frame, as shown below. The area of the wooden frame is $18x$ square units, and the area of the mirror is $32x$ square units. Let C represent the center of the mirror. If $CE = y$, find DE , the width of the wooden frame.



- A. $0.8y$ B. $0.36y$ C. $0.64y$ D. $0.2y$ E. $0.6y$ F. None of these
17. Find the equation of the line that is tangent to the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ at the point $(7, 1)$.
- A. $4x + 3y = 25$
B. $10x + y = -69$
C. $4x - 3y = 25$
D. $3x - 4y = 17$
E. $4x + 3y = 31$
F. None of these

18. The solid below is made of a cylinder and a hemisphere. The cylinder's height is equal to its radius. If the area of the circle is 18 square units, find the total surface area of the solid, in square units.

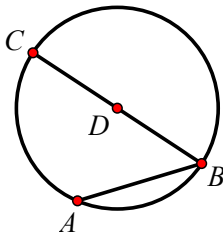


- A. 54
- B. 72
- C. 90
- D. 126
- E. 108
- F. None of these

19. Triangle DEF has vertices $D\left(\frac{1}{2}, \frac{1}{3}\right)$, $E\left(\frac{4}{5}, \frac{5}{6}\right)$, and $F\left(\frac{4}{5}, \frac{1}{3}\right)$. Find $\sin(D)$.

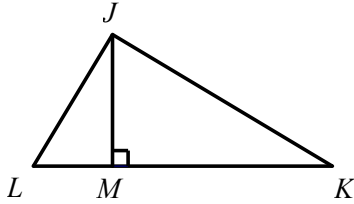
- A. $\frac{5}{17}$
- B. $\frac{3\sqrt{34}}{34}$
- C. $\frac{\sqrt{34}}{10}$
- D. $\frac{5}{3}$
- E. $\frac{5\sqrt{34}}{34}$
- F. None of these

20. Circle D is shown below. If $DB = 30$ cm and the arclength of \widehat{AC} is 8π cm, find $m\angle ABC$.



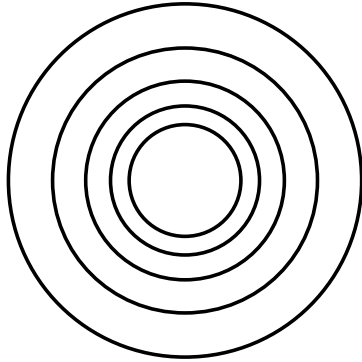
- A. 156°
- B. 48°
- C. 96°
- D. 132°
- E. 24°
- F. None of these

21. Find the perimeter of right triangle JKL , given that $LM = 4$ and $MK = 5$.



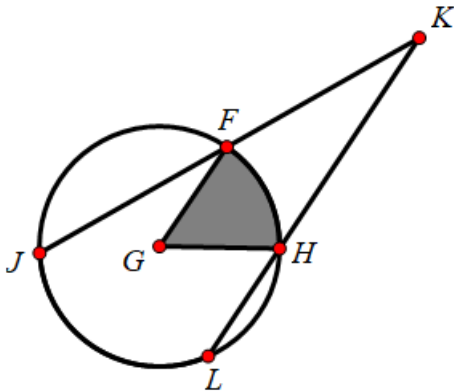
- A. $9 + 5\sqrt{5} + 2\sqrt{29}$ B. $3\sqrt{5} + 15$ C. $5 + 5\sqrt{5}$
 D. $9 + \sqrt{105} + 4\sqrt{6}$ E. $15 + 5\sqrt{5}$ F. None of these
22. Rania has sticks that snap together on the ends to form polygons. She puts four sticks into a hat, two of which are 3 inches long and two of which are 5 inches long. She closes her eyes, draws one stick out the hat and puts it on the table; this stick becomes \overline{AB} . She randomly draws another stick out of the hat; this stick gets snapped to the first stick to become \overline{BC} . Rania randomly draws another stick out of the hat; this stick becomes \overline{CD} as it is snapped onto \overline{BC} . She then draws the last stick from the hat, which becomes \overline{DA} and completes quadrilateral $ABCD$. If all sticks in the hat at a given time are equally likely to be chosen, what is the probability that Rania's quadrilateral is a parallelogram?
- A. $\frac{1}{6}$ B. $\frac{1}{4}$ C. $\frac{1}{3}$ D. $\frac{1}{2}$ E. $\frac{2}{3}$ F. None of these
23. Segment \overline{BC} is dilated about center $A(5, 7)$ with a scale factor of 50%, resulting in segment $\overline{B'C'}$. If point B has coordinates $(-5, -1)$, and the midpoint of \overline{BC} has coordinates $(-3, 0.5)$, find the coordinates of C' .
- A. $(0, 3)$
 B. $(-4, -0.5)$
 C. $(2, 4.5)$
 D. $(1, 3.75)$
 E. $(-1, 2)$
 F. None of these

24. Carly makes a decorative plate with a design of five concentric circles. The outer circumference of the plate is 50π cm. Each circle's area is 20% smaller than the area of the circle immediately surrounding it. Find the circumference of the smallest circle on the plate, in centimeters.



- A. $\frac{512\pi}{25}$
- B. 256π
- C. 2π
- D. 32π
- E. 64π
- F. None of these

25. Circle G is shown below. If $GF = 10$ cm, $m\angle K = 34^\circ$, and the area of the shaded sector is 20π square centimeters, find $m\widehat{JL}$.



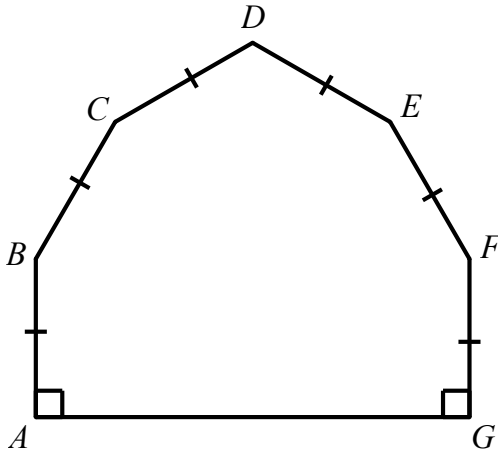
- A. 72°
- B. 104°
- C. 106°
- D. 140°
- E. 144°
- F. None of these

Continued on the next page...

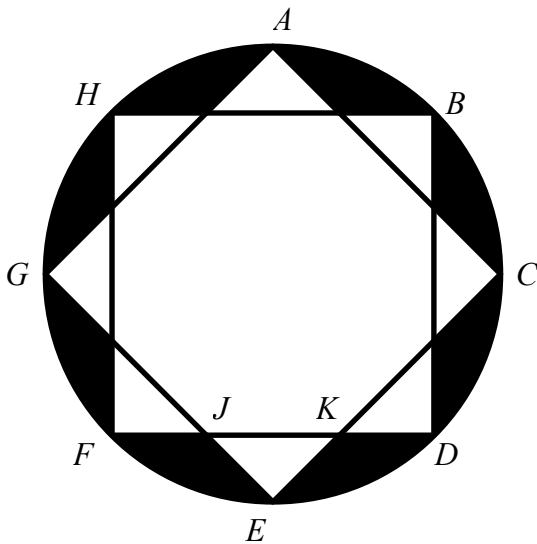
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Questions 26-28 are write-in answers rather than multiple choice. Write the answer to each question on the answer sheet in the space provided. All irrational answers (containing radicals or π) should be left as exact answers rather than decimal approximations. All radicals and fractions should be written in simplest form, with no radicals in the denominator. Do not write any units on the answer sheet.

26. Find the area of the following barn-shaped polygon, given that $m\angle D = 120^\circ$, $m\angle B = m\angle F = 150^\circ$, and $BC = 12$. Find the area of $ABCDEFG$. ($ABCDEFG$ is reflectionally symmetric.)

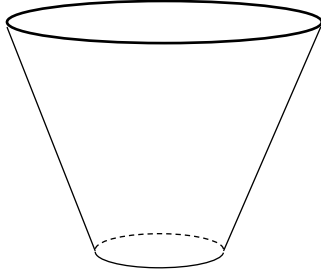


27. An eight-pointed star has been drawn by starting with a circle, placing eight equally-spaced points around the circumference of the circle (points A through H), and then drawing segments as shown. If $JK = 8$, find the area of the white star-shaped region. Do not include the area that is shaded.



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28. The cup below is a truncated cone, with a height of 8 cm, a top diameter of 18 cm and a bottom diameter of 6 cm. If it is filled with water to half of its volume, what would the depth of the water be?



END OF EXAM

