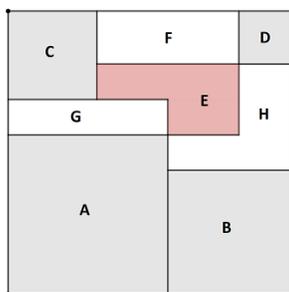


Algebra 1 Exam - University of Houston Math Contest January 30, 2021

1) Kris runs half of the distance to school averaging 6mph. He jogs the rest of the way to school averaging 4mph, and the whole trip takes him 25 minutes. How many minutes will it take him to run the same way home if he averages 8mph the whole way?

- a) 18 min b) 16 min c) 17 min d) 20 min e) 15 min f) 10 min

2) The squares labeled A, B, C, D have areas of 81, 49, 25 and 9 units², respectively. These four squares along with the other non-overlapping rectangles and polygons labeled E, F, G and H are arranged to form a large square as shown. What is the area of the polygon E?



- a) 92 units² b) 24 units² c) 32 units² d) 50 units² e) 16 units² f) 28 units²

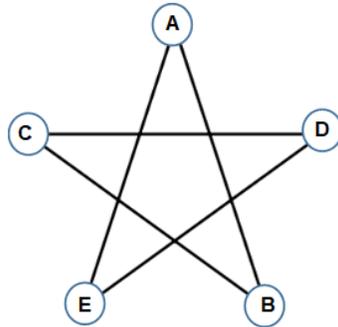
3) Megan is taking a very difficult biology course, and her previous exam scores were 95, 94, 89, and 79. What is the lowest score Megan can get on her next test to average an A in the class (90%)?

- a) 95 b) 91 c) 92 d) 90 e) 93 f) 94

4) William has a digital coin bank, where he collects only dimes and quarters. After he deposited a handful of coins for the New Year, the digital screen showed the value of \$10.00. Knowing that the bank contains more than 10 coins of each type, what is the minimum number of coins worth of \$10.00 the bank can have?

- a) 52 b) 49 c) 46 d) 48 e) 50 f) 47

5) In the five-sided star shown, the letters A, B, C, D, and E are replaced by the numbers 13, 15, 16, 17, and 19, although not necessarily in this order. The sums of the numbers at the ends of the line segments AB, BC, CD, DE, and EA form an arithmetic sequence, although not necessarily in that order. What is the middle term of this arithmetic sequence?



- a) 38 b) 36 c) 35 d) 32 e) 30 f) 31

6) A bag contains blue, red, green, black, and white marbles. $\frac{1}{3}$ are red, $\frac{1}{4}$ are blue, and $\frac{1}{6}$ are black. Of the remaining, 7 marbles are white, and 8 marbles are green. If two marbles are drawn without the replacement of the first, what is the probability that both are black or both are blue?

- a) $\frac{3}{118}$ b) $\frac{1}{36}$ c) $\frac{1}{24}$ d) $\frac{7}{118}$ e) $\frac{5}{59}$ f) $\frac{13}{144}$

7) Memorial Middle School has the same number of boys and girls. Three-fourths of the girls and two-thirds of the boys went on a field trip. What fraction of the students were girls?

- a) $\frac{9}{17}$ b) $\frac{6}{13}$ c) $\frac{8}{17}$ d) $\frac{7}{13}$ e) $\frac{2}{3}$ f) $\frac{1}{2}$

8) A **palindrome** is a number that is read the same backwards as forwards, for instance 12321. How many 5-digit palindromes **with different neighboring digits** are there? (Note that 01210 is not considered a 5-digit number and the palindrome 52525 has different neighboring digits.)

- a) 900 b) 729 c) 648 d) 810 e) 504 f) 720

9) Diane is an artist and she wants to buy two sets of professional paint acrylic markers that are on sale. One of them has a retail price of \$200, and it is now 30% off. The other set has retail price \$160, and it is 40% off. She also has a coupon for 20% off the total price she pays. How much does Diane pay if she buys both the sets?

- a) \$273.60 b) \$188.80 c) \$196 d) \$128 e) \$164 f) \$288

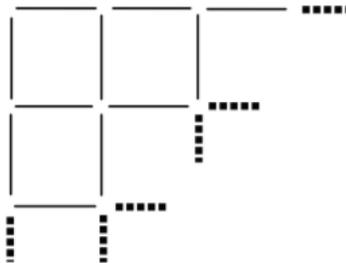
10) Alice and Bob are coloring some pictures. If it takes 5 minutes for Alice to color 4 pictures alone and 3 minutes for both children to color 4 pictures together, how many complete pictures can Bob color in 15 minutes ?

- a) 7 b) 8 c) 5 d) 6 e) 9 f) None of the above.

11) If $a \star b = \frac{\text{Greatest Common Divisor of } a \text{ and } b}{\text{Least Common Multiple of } a \text{ and } b}$, then evaluate $180 \star 594$.

- a) $\frac{1}{330}$ b) $\frac{1}{660}$ c) $\frac{1}{297}$ d) $\frac{1}{165}$ e) $\frac{1}{625}$ f) $\frac{1}{110}$

12) Toothpicks are used to make a grid that is 50 toothpicks long and 25 toothpicks high. How many toothpicks are used altogether?



- a) 2, 626 b) 2, 575 c) 1, 250 d) 1, 300 e) 1, 326 f) 1, 275

13) For a positive integer n , the factorial notation $n!$ represents the product of the integers from n to 1. (For example, $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$.) Which of the following numbers is a perfect square?

- a) $\frac{29! \cdot 30!}{2}$ b) $\frac{33! \cdot 34!}{2}$ c) $\frac{28! \cdot 29!}{2}$ d) $\frac{32! \cdot 33!}{2}$ e) $\frac{31! \cdot 32!}{2}$ f) $\frac{30! \cdot 31!}{2}$

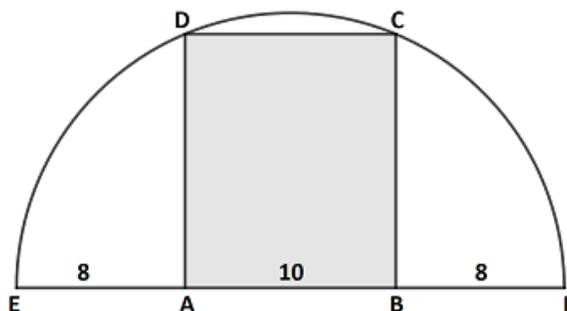
14) Let $f(x)$ be a linear function. Given $f(6) - f(2) = 12$, find the value of $f(12) - f(2)$.

- a) 48 b) 36 c) 15 d) 30 e) 28 f) 20

15) David drives from his home to the airport to catch a flight. He drives 45 miles in the first hour, but realizes that he will be 1 hour late if he continues at this speed. He increases his speed by 15 miles per hour for the rest of the way to the airport and arrives 30 minutes early. How many miles is the airport from his home?

- a) 405 b) 360 c) 315 d) 225 e) 135 f) 270

16) Rectangle ABCD is inscribed in a semicircle with diameter \overline{EF} as shown in the figure below. Let $AB = 10$ and let $EA = BF = 8$. What is the area of ABCD?

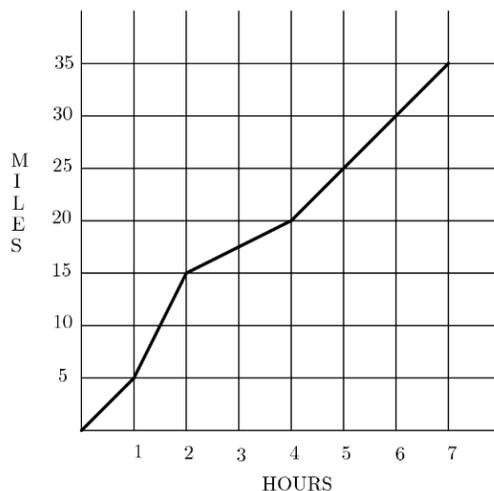


- a) 200 b) 120 c) 100 d) 160 e) 180 f) 140

17) What is the sum of all real numbers for which $(x^2 - 14x + 47)^2 = 4$?

- a) 28 b) 47 c) 14 d) 42 e) 21 f) 33

18) John takes a long bike ride on a hilly highway. The graph indicates the miles traveled during the time of his ride. What is John's average speed for his entire ride in miles per hour?



- a) 4 b) 2.5 c) 5 d) 3.5 e) 4.5 f) 3

19) Lizzie skip-counts out loud. The first three numbers she says can be represented by the expressions n , $n + 9$, and $4n$, in that order. If Lizzie continues her pattern, what is the next number she will say?

- a) 15 b) 36 c) 42 d) 24 e) 33 f) 60

20) In the following Zig-Zag puzzle, each bottom number is either multiplied or divided by the top number to its right according to the following rule:
 If the bottom number is divisible by the top number to its right, then divide, otherwise multiply. The result is the next bottom number to the right. Following this rule, find $x + y$ from the following puzzle:



- a) -15 b) -16 c) 31 d) 1 e) -31 f) -1

21) Andy multiplies four different integers. The absolute value of each integer is less than 10. Find the least possible product of the four integers.

- a) $-5, 184$ b) $3, 024$ c) $-4, 536$ d) $-3, 024$ e) $4, 536$ f) 0

22) In a class of 10 monsters, removing the shortest monster increases the average height by 2 inches. Removing the tallest monster decreases the average height from 28 to 22. What is the range of the monster heights?

- a) 72 b) 60 c) 28 d) 82 e) 88 f) 10

23) The heights in centimeters of 7 flowers are distinct positive integers with a mean of 80 and a median of 81. What is the greatest possible height one flower can have?

- a) 100 b) 324 c) 308 d) 181 e) 84 f) 320

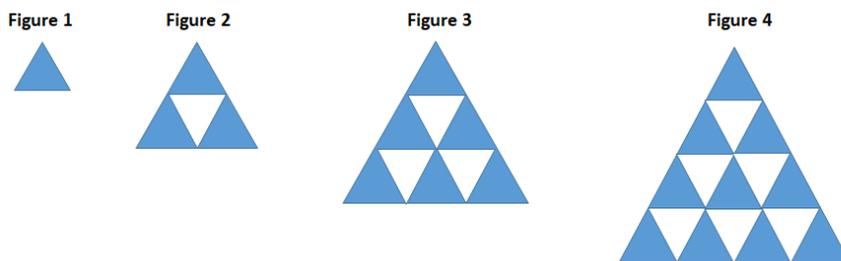
24) Niki plays a game with a standard die. Every time she rolls a prime number she gets 45 points. Every time she does not roll a prime number she loses 12 points. At the end of a game, she has 0 points. What is the smallest number of rolls she could have made?

- a) 19 b) 15 c) 14 d) 17 e) 16 f) 18

25) Captain Black Jack has a satchel of gold and silver coins. The gold coins are worth \$50 each, while the silver coins are worth \$12 each. The average value of a coin in the satchel is \$30. What is the least number of coins that could be in Black Jack's satchel?

- a) 11 b) 21 c) 17 d) 15 e) 19 f) 13

26) If the pattern below continues, what percent of **Figure 50** will be shaded?



- a) 55% b) 52% c) 51.5% d) 51% e) 50% f) 50.5%

27) If the largest prime factor of $13! + 15! + x!$ is 421, find the smallest possible value of x .

- a) 17 b) 11 c) 15 d) 23 e) 21 f) 13

28) What is the last digit of $28^{2021} + 19$?

- a) 8 b) 0 c) 4 d) 5 e) 7 f) 3

29) If the minimum value of $f(x) = x^2 + 2bx + 2c^2$ is greater than the maximum value of $g(x) = -x^2 - 2cx + b^2$, then which of the following must be true?

- a) $c = b$ b) $c > \sqrt{2} \cdot b$ c) $\sqrt{2} \cdot c > b$ d) $\sqrt{2} \cdot |c| > |b|$ e) $|c| > \sqrt{2} \cdot |b|$ f) $c < b$

30) Find the product of all integer values of k for which the equation $x^2 - kx + 2k = 3$ has no real roots.

- a) 15 b) 20 c) 60 d) 12 e) 720 f) 120

31) Let $f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ be a polynomial function. Each of the coefficients a_i , $0 \leq i \leq n$, of this polynomial is either 0 or 1. If $f(3) = 118$, find $f(4)$.

- a) 219 b) 337 c) 119 d) 340 e) 318 f) 237

32) If the area of the triangle bounded by the perpendicular lines $x - ay = 0$, $10x + 5y = b$ and the x -axis in the first quadrant is 5 units², find $a + b$.

- a) 48 b) 32 c) 15 d) 52 e) 27 f) 18

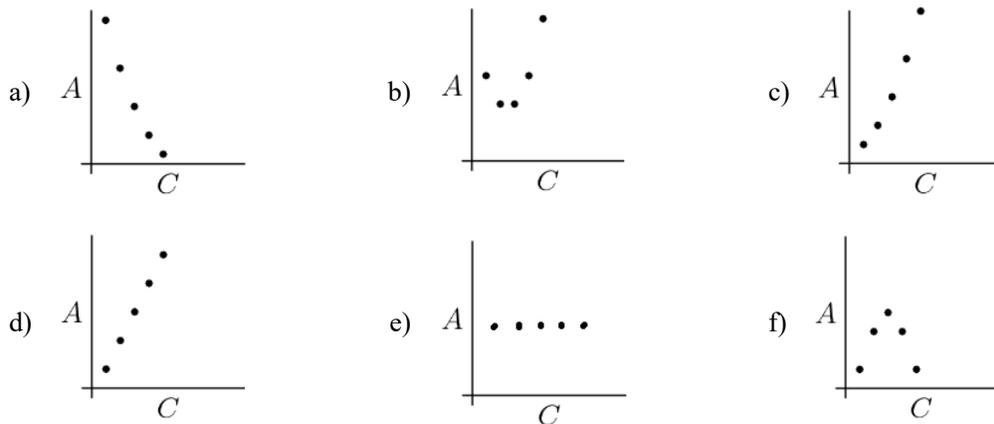
33) Let R be the region bounded by the lines $2x + y = 4$ and $2x + y = 10$ in the first quadrant, and let A be an ordered pair (x, y) from this region. What is the probability that the first coordinate of the point A is at most 2?

- a) $\frac{6}{7}$ b) $\frac{1}{3}$ c) $\frac{3}{7}$ d) $\frac{2}{3}$ e) $\frac{4}{7}$ f) $\frac{1}{7}$

34) The school bus comes to a stop near your house every day at a random time between 7:45 and 7:50. You arrive at the bus stop at a random time between 7:40 and 7:45 every day and wait until the bus comes. What is the probability that you wait less than 4 minutes for the bus to arrive?

- a) 30% b) 25% c) 32% d) 28% e) 35% f) 38%

35) Amanda draws five circles with radii 1, 2, 3, 4 and 5. Then for each circle she plots the point (C, A) , where C is its circumference and A is its area. Which of the following could be her graph?



36) What is the remainder when $(1 + 2 + 3 + \dots + 48)^{49}$ is divided by 50?

- a) 49 b) 26 c) 1 d) 42 e) 24 f) 36

37) What is the 1,896,253rd digit in the decimal expansion of $\frac{1}{41} = 0.\overline{02439}$?

- a) 9 b) 4 c) 0 d) 3 e) 5 f) 2

38) If $\sqrt{32 + 8\sqrt{7}} = a + \sqrt{b}$ where a and b are both integers, find $a + b$.

- a) 24 b) 9 c) 28 d) 11 e) 30 f) 14

39) Suppose that $a \circ b$ means $3a - b$. What is the value of x if $2 \circ (5 \circ x) = 1$?

- a) 14 b) 10 c) $\frac{1}{10}$ d) 2 e) 8 f) $\frac{10}{3}$

40) Keith wants to mark all of the original prices in his store down to half-off. The prices are already marked down by 20%. What additional percent should be discounted?

- a) 33.3% b) 37.5% c) 62.5% d) 30% e) 50% f) 66.6%

41) Everyone seated in the UH Agnes Arnold auditorium is either a student or a professor. The ratio of students to professors is 5:1, and the ratio of filled seats to empty seats is 4:1. What is the ratio of the professors to the empty seats in the auditorium?

- a) 1:3 b) 2:15 c) 1:15 d) 2:3 e) 3:2 f) 15:2

42) If $\frac{17x - 5}{6}$ and $\frac{14x + 4}{9}$ are integers where $x > 1$ is a positive integer, then find the least integer value of $\frac{17x - 5}{6}$.

- a) 52 b) 53 c) 32 d) 30 e) 31 f) 28

43) If x and y are positive integers such that $xy + x + y = 71$ and $x^2y + xy^2 = 880$, then find $x^2 + y^2$.

- a) 36 b) 256 c) 91 d) 809 e) 146 f) 136

44) After the wall clock shows 4:10pm, how many minutes does it take to get an angle of 45° between the long hand and the short hand of the clock for the 2nd time? (Note that this wall clock is a 12-hour traditional one.)

- a) 10 min b) 23.6 min c) 20 min d) 5 min e) 30 min f) 3.6 min

45) Logan has an old analogue watch which runs 30 seconds behind every hour. After he sets his watch correctly, how many days will it take for the watch to show the right time again?

- a) 45 b) 75 c) 60 d) 105 e) 120 f) 90