Algebra 2 Exam - University of Houston Math Contest January 30, 2021

1) Let a be the minimum value of the range of the function f(x) = ||x+3|+2| and let b be the maximum value of the domain of the function $g(x) = \sqrt{-64 - x^3}$. If we define $h(y) = a^{by}$, then what is the value of h(2)?

a) 2^{16} b) 256 c) $\frac{1}{8}$ d) $\frac{1}{128}$ e) $\frac{2\sqrt{2}}{3}$ f) $\frac{1}{256}$

2) Find the area of the region enclosed by the curves y(x) = |x+3| + 2 and y(x) = -|x+3| + 8.

3) Suppose
$$g(x) = \frac{ax+b}{cx+d}$$
 and that its inverse $g^{-1}(x) = \frac{3+2x}{5x+1}$. Find one possible value of $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}$.
a) $\frac{1}{30}$ b) $-\frac{29}{15}$ c) $\frac{29}{15}$ d) $-\frac{29}{30}$ e) $\frac{19}{30}$ f) $\frac{61}{30}$

4) What is the domain of the function f(x) = log(log(log(20 - x))) + log(log(x))?
a) (-∞, 20) b) (-∞, 20] c) (1,∞) d) (10,∞) e) (1,10) f) (-∞, 0)

5) Let *I* be the domain of the function $f(x) = \sqrt{100 - (x+5)^2}$. Determine the percentage of the interval *I* for which the inequality $x > 4 - \frac{7}{x+4}$ is satisfied.

a) 12% b) 15% c) 30% d) 10% e) 9% f) None of the above

6) Let f(x) = x - 7, $g(x) = \sqrt{x}$, and $h(x) = 2^x - 4$. Find the domain of $g \circ f^{-1} + h^{-1} \circ f^{-1}$.

a) $(-7,\infty)$ b) $(-3,\infty)$ c) $(-11,\infty)$ d) $[-7,\infty)$ e) $(0,\infty)$ f) $(-\infty,11]$

7) Let f(x) be defined by $f(x) + 3f\left(\frac{1}{x}\right) = 5x$ for all real numbers $x \neq 0$. Find f(5).

a)
$$\frac{14}{5}$$
 b) 22 c) $-\frac{11}{5}$ d) $\frac{11}{8}$ e) $-\frac{11}{4}$ f) $\frac{22}{10}$

8) Let $f(x) = \frac{x^2+5}{x+3}$. For what positive value of x is f(x+2) = 2x+2?

a)
$$\frac{-3 + \sqrt{5}}{2}$$
 b) 2 c) $-4 + \sqrt{15}$ d) 0 e) $\frac{-3 + \sqrt{13}}{2}$ f) None of the above

9) The line with slope m = 1 and y-intercept b = 2, intersects the circle of radius r = 4, centered at the origin, at two points. What is the distance between these two points?

a) 10 b) $8\sqrt{5}$ c) $\sqrt{28}$ d) $2\sqrt{7}$ e) $2\sqrt{14}$ f) None of the above

10) I bought a certain number of books at a price of 5 books for \$60. I kept a third of the books and sold the rest at a price of 4 books for \$90. If I made a profit of \$90, how many books did I buy?

a) 20 b) 30 c) 7 d) 50 e) 3 f) 10

11) Sam does the shopping for his family. This month he went three times to the grocery store. The first time he paid \$17.50 for 3 bags of tortillas, 1 pound of sugar, and 1 gallon of milk. The second time he bought 2 bags of tortillas, 2 pounds of sugar and 2 gallons of milk and paid a total of \$21. At the end of the month his family had a party for his sister's birthday, so Sam bought 5 bags of tortillas, 2 pounds of sugar, and 3 gallons of milk for a total of \$36.50. His mother was surprised by the rising price of groceries, and concluded that the price of a gallon of milk probably went up from \$4.90 compared to the previous month. Was she right? How much was Sam paying for a gallon of milk?

a) \$4.91 b) \$4.00 c) \$5.11 d) \$4.50 e) \$5.83 f) \$5.00

12) A movie theater can hold 300 people. The price for a regular ticket is \$12, while children pay \$8. On the day the movie 'Frozen' opened, the theater grossed \$22,576 on that movie alone. If the theater had 8 showings of the movie, how many children watched the movie?

a) 1556 b) 2400 c) 5044 d) 300 e) 844 f) $\frac{389}{2}$

13) Megan and her mom Holly are celebrating their birthday today. Eight years ago Holly's age was three times that of Megan's, but in four years Megan will be $\frac{5}{9}$ Holly's age. What will Megan's age be in ten years?

a) 26 b) 25 c) 16 d) 32 e) 42 f) 30

14) A large group of friends rented a bus to go to the beach. They decided to split the total cost of the bus ride evenly among them. If 10 more people would have joined the trip, each person would have paid \$5 less for the bus. On the other hand, if 6 people would have stayed home, then each person would have paid \$5 more. How many people went to the beach?

a) 25 b) 26 c) 60 d) 32 e) 30 f) 42 15) Find the value of $(a + b)^3$ if $2(a^3 + b^3) + a^2b + b^2a = 336$ and $a^3 + b^3 - 3(a^2b + b^2a) = 77$. a) 216 b) 64 c) 336 d) 729 e) 233 f) 512

16) Rambo has a bakery that specializes in chocolate chip and sugar cookies. A bag of 6 chocolate chip cookies sells for \$10, while a bag of 8 sugar cookies for \$10. To cover his expenses he has to sell at least \$300 worth of cookies, but he has ingredients to make at most 200 cookies. Which of the following options will allow him to pay for his expenses?

a) Making 120 chocolate chip cookies and 80 sugar cookies

b) Making 100 chocolate chip cookies and 100 sugar cookies

c) Making more than 100 sugar cookies

d) Making 90 chocolate chip cookies and 120 sugar cookies

e) Making 80 chocolate chip cookies and 120 sugar cookies

f) None of the above

17) A rectangle is inscribed inside a circle of radius r. The perimeter of this rectangle is P = 16 and its area is A = 15. What is the value of r?

a) 5 b) 1 c)
$$\frac{\sqrt{34}}{2}$$
 d) 10 e) $\sqrt{15}$ f) 3

18) Two parabolas intersect at the points (2, 0) and (-3, -15). Their axis of symmetry are the vertical lines passing through x = 1 and x = -2. Let b_1 and b_2 be the y component of their vertex. What is the value of $b_1 + b_2$?

a) -10 b) 15 c) 10 d) -15 e) 17 f) 3

19) The parabolas $y = -x^2 + 5x$ and $y = (x+3)^2 - 12$ intersect at two points. Let L be the line that passes through these points. Which of the following points is also on the line L?

a)
$$\left(3, \frac{55}{10}\right)$$
 b) $\left(2, \frac{19}{2}\right)$ c) $(1, -4)$ d) $\left(-2, \frac{19}{2}\right)$ e) $(-1, -70)$ f) $(0, -15)$

20) In the given figure, let *B* be the point on the line segment connecting the points P = (0, 16) and Q = (8, 0). What is the maximum area of the rectangle *ABCD* if the point D = (0, -2)?



21) Find the sum of all elements in the solution set of both of the equations,

$$y + \sqrt{y-4} = 4$$

 $\sqrt{2x-1} - \sqrt{x-4} = 2$
a) 23 b) 17 c) 18 d) 22 e) 10 f) 9

22) If p is a positive integer, what is the sum of the roots of the equation $(3x - 5)^2 - p = 0$?

a) 10 b) 25 - p c) -30 d) 9 e) $\frac{10}{3}$ f) $-\frac{30}{18}$

23) When $2x^4 + 7x^2 - 15$ is factored completely with integer coefficients, the sum of the factors is:

a) 2x + 7 b) $3x^2 + 2$ c) $2x^2 - 15$ d) $x^2 + 4$ e) $3x^2 + 7$ f) None of the above

24) Find the sum of the real solutions to the equation $\sqrt{x} = \frac{6}{5 - \sqrt{x}}$.

25) If b > 6, solve the following inequality $\frac{x^2 + x - 6}{x - b} \ge 0$.

a) $[0,\infty)$ b) (-3,2) c) $[-2,3] \cup (b,\infty)$ d) (b,∞) e) $[-3,2] \cup (b,\infty)$ f) None of the above

26) If
$$f(x) = \frac{x^4 - x^3 + x^2 - x + 1}{-x}$$
 and $i = \sqrt{-1}$, find $f(2f(i))$.
a) i b) $\frac{6 - 13i}{2}$ c) $2i$ d) $\frac{13i - 6}{2}$ e) $\frac{6 + 3i}{2}$ f) $\frac{10 - 13i}{2}$

27) Ravi's age is three years more than twice his sister's age. If the sum of their ages is at most 18, then what is the maximum possible age of Ravi's sister?

a) 5 b) 15 c) 18 d) 9 e) 2 f) None of the above

28) If
$$a + bi = \frac{3-i}{1+i}$$
, then find a, b .
a) $a = \frac{3}{2}, b = -2$ b) $a = 1, b = 1$ c) $a = 1, b = -2$
d) $a = 3, b = -4$ e) $a = 0, b = 2$ f) $a = 1, b = 2$

29) Which of the following expressions is equivalent to $\log_8(32x) - \log_4(8x) + \log_2(x)$?

a)
$$\frac{1}{6}\log_2(2x^5)$$
 b) $\log_2(3x^{\frac{1}{3}})$ c) $3\log_2(8x^2)$ d) $\frac{1}{6}\log_2(\frac{2}{x})$ e) $\log_2(2x^5)$ f) None of the above

30) What is the number of distinct real values of x which satisfy the equation $(x^2 - 5x + 5)^{x^2 - 9x + 20} = 1$? a) 0 b) 3 c) 4 d) 2 e) 1 f) None of the above

31) If $\log_{16} 5 = p$ and $\log_5 7 = q$, then find $\log_{14} 7$ in terms of p and q.

a)
$$\frac{7pq}{1+7pq}$$
 b) $\frac{2pq}{p+q}$ c) $\frac{2pq}{1+2pq}$ d) $q+4p$ e) $1-4pq$ f) $\frac{4pq}{1+4pq}$

32) Given that
$$\log_3 \sqrt[4]{\frac{1}{81}\sqrt[8]{x\sqrt{\frac{729}{81^{0.5}}}}} = -\frac{7}{8}$$
. Find the sum of the digits of the value of x .
a) 3 b) 16 c) 9 d) 18 e) 6 f) 17

University of Houston Math Contest 2021 https://mathcontest.uh.edu 33) Find the roots of

$$|x-7|^2+2|x-7|=24$$

a) $\{1, 11\}$ b) $\{11, 3\}$ c) $\{4, 10\}$ d) $\{6, 4\}$ e) $\{1, 13\}$ f) None of the above

34) What is the sum of the squares of the real and complex solutions of $x^4 + 4x^3 + 7x^2 + 28x = 0$?

a) 2 b) 4 c) 30 d) 17 e) -4 f) 28

35) Suppose the weight of a culture of bacteria doubles daily. If the weight of the culture at the end of b days is K grams, what is the weight of the culture at the end of b - 3 days?

a)
$$K$$
 b) $\frac{K}{16}$ c) $\frac{K}{8}$ d) $\frac{K}{3}$ e) K^{b-3} f) $8K$

36) If (-1,5) is a point on the graph of y = f(x), then the graph of y = f(x-3) - 2 contains the point (c,3) where c equals what value?

a) 4 b) 1 c) 3 d) -1 e) -c f) 2

37) Solve the equation $2^{-4x} + 0.25a^2 = a2^{-2x}$ for *x*.

a)
$$\frac{1}{2}\log_2\left(\frac{2^{-2}-a}{\sqrt{a}}\right)$$
 b) $\frac{1-\log_2 a}{2}$ c) $-\log_2(2-\sqrt{a})$ d) $\frac{a}{2}$ e) 2 f) 4

38) For each real number x, let f(x) be the minimum of the numbers 2x + 1, x - 1, -2x + 5. Find the maximum value of f(x).

a) 3 b) 2 c) 10 d) -3 e) 0 f) 1

39) How many ordered pairs (m, n) of positive integers are solutions to $\frac{10}{m} - \frac{3}{n} = 1$?

a) 5 b) 4 c) 2 d) 3 e) 0 f) 1

40) Which of the following expressions is different from the other 3?

I.
$$\frac{1}{2} - \frac{i+c}{i-1}$$
 II. $\frac{c}{2} - \frac{i+ci}{2}$ III. $\frac{i}{2} - \frac{c}{i+1}$ IV. $\frac{-1 + (2c+1)i}{2(1+i)}$

a) II b) All are the same c) I d) IV e) None are the same f) III

41) Find the equation of the line tangent to the circle $x^2 + 8x + y^2 + 6y = 0$ at the point (-1, 1).

a)
$$y = \frac{1}{4} - \frac{3x}{4}$$

b) $y = x + 2$
c) $y = \frac{x}{2} + \frac{1}{4}$
d) $y = -x - 2$
e) $y = \frac{4x}{3} + \frac{7}{3}$
f) $y = -\frac{5x}{6} + \frac{1}{6}$

42) The first two of three consecutive multiples of 7 sum to 1897. What is the largest of these three numbers?

a) 98 b) 910 c) 91 d) 271 e) 980 f) 959

43) Find the value of the continued fraction

$$3 + \frac{1}{4 + \frac{1}{3 + \frac{1}{4 + \dots}}}$$

a) $\frac{3}{2}$ b) $\frac{12 - \sqrt{192}}{8}$ c) $\frac{3}{2} + \sqrt{3}$ d) $\frac{3}{2} + \frac{\sqrt{6}}{2}$ e) $\frac{3}{2} + \frac{\sqrt{3}}{2}$ f) None of the above

44) Find the area of the region consisting of all the points in the plane that satisfy $1 \le |x| + |y| \le 2$.

a) 16 b) 10 c) 6 d) 2 e) 5.5 f) 8