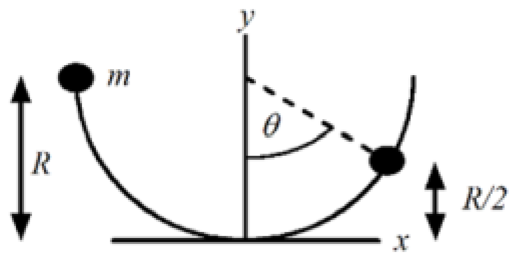
Physics Exam - University of Houston Math Contest
January 30, 2021

Unless otherwise specified, please use g as the acceleration due to gravity at the surface of the earth. Vectors $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ are unit vectors along x, y, and z, respectively, in a normal Cartesian coordinate system. Let G be the universal gravitational constant. To simplify calculations, you may use $g = 10\,\mathrm{m/s}^2$.

Questions 1 - 3 pertain to a mass m that slides on a semicircular wire as shown. The coefficient of kinetic friction between the wire and mass m is μ. Mass m is released from rest at a height R above the bottom of the hemisphere and stops at a height $R/2$.



1) The work done by friction during by friction during this process is given by

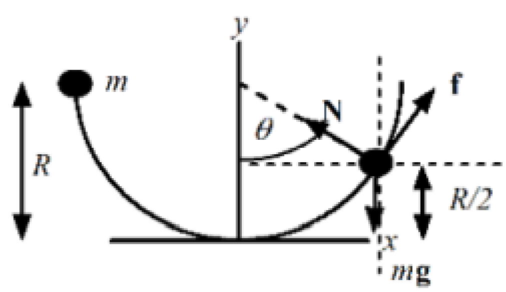
a) mgR b) $mgR/2$ c) $-mgR/2$ d) 0 e) $-mgR$ f) none of the other answers provided

2) The magnitude of the average force due to friction is given by

a) $\dfrac{mg}{\pi + 2\cos^{-1}\left(\frac{1}{2}\right)}$ b) $\dfrac{mg}{\pi - 2\cos^{-1}\left(\frac{1}{2}\right)}$ c) 0 d) $\dfrac{mg}{\pi - \cos^{-1}\left(\frac{1}{2}\right)}$ e) $\dfrac{mg}{\pi + \cos^{-1}\left(\frac{1}{2}\right)}$

f) none of the other answers provided

3) In order for the object to remain at rest at the height $R/2$, the relationship between the force due to friction f, the normal force of contact N between mass m and the wire, the force due to gravity mg, and θ is given by



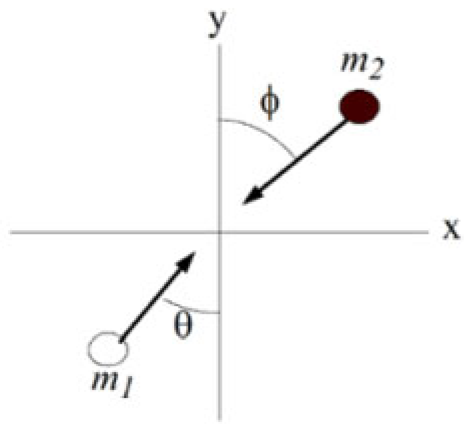
a) $f\sin(\theta) + N\cos(\theta) - mg = 0$ b) $f\cos(\theta) + N\sin(\theta) - mg = 0$ c) $f\sin(\theta) + N\sin(\theta) - mg = 0$

d) $f\cos(\theta) + N\cos(\theta) - mg = 0$ e) none of the other answers provided

Questions 4 - 6 pertain to the two-dimensional collision between two disks, m_1 and m_2, on an air table without friction. They collide at the origin of the coordinate system shown. The two disks stick together after the collision. You wish to find the $x-$component of the velocity, V_x , and $y-$component of the velocity, V_y , of the combined mass after the collision and the angle β that the velocity makes with respect to the positive x-axis. Let

$$m_1 = \frac{7}{5}m_2,\ \cos(\theta) = \frac{4}{5},\ \cos(\phi) = \frac{3}{5},\ \text{and } v_2 = \frac{5}{4}v_1$$

where v_1 and v_2 are the magnitudes of the velocities of m_1 and m_2, respectively.



4) V_x is given by

a) $-\left(\frac{1}{15}\right)v_2$ b) $\left(\frac{1}{15}\right)v_2$ c) $\left(\frac{1}{15}\right)v_1$ d) $-\left(\frac{1}{15}\right)v_1$ e) none of the other answers provided

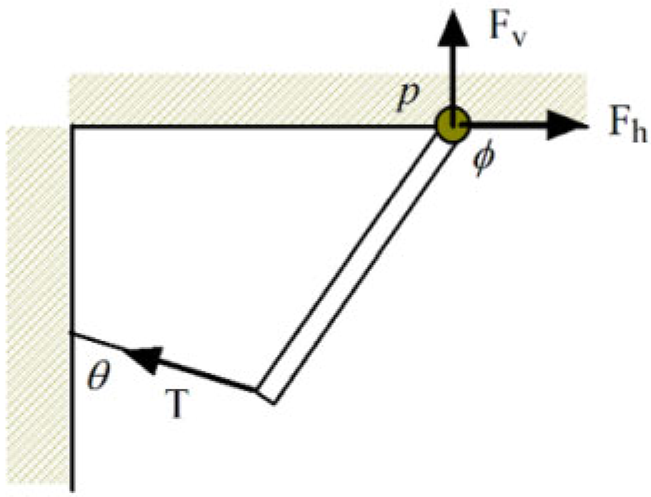
5) V_y is given by

a) $\left(\frac{2}{15}\right)v_2$ b) $\left(\frac{1}{4}\right)v_1$ c) $-\left(\frac{2}{15}\right)v_2$ d) $-\left(\frac{1}{4}\right)v_1$ e) none of the other answers provided

6) The angle β is given by

a) $\pi + \tan^{-1}\left(\frac{16}{37}\right)$
b) $\pi + \tan^{-1}\left(\frac{37}{16}\right)$
c) $\pi + \tan^{-1}\left(-\frac{16}{37}\right)$
d) $\pi + \tan^{-1}\left(-\frac{37}{16}\right)$
e) cannot be determined
f) none of the other answers provided

Questions 7 - 9 pertain to a student studying torque using a configuration that is shown in the figure. A bar is suspended from a pivot p and a string attaches the bar to the wall on the other end. The length of the bar is L, its center of mass is located a distance l_{cm} from the pivoted end, and the tension in the string is T. The angles θ and ϕ are shown in the figure. You wish to calculate the horizontal and vertical components of the force the pivot exerts on the bar, F_h and F_v, respectively, and the bar's mass m. Use a standard coordinate system with $+x$ to the right and $+y$ up the page.



7) The expression for F_h is given by

a) $F_h = T\cos(\theta)$
b) $F_h = T\sin(\theta)$
c) $F_h = T\cot(\theta)$
d) $F_h = T\tan(\theta)$
e) none of the other answers provided

8) The expression for m is given by

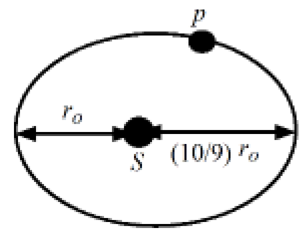
a) $\dfrac{TL\cos(\theta)}{gl_{cm}\cos(\theta+\phi)}$ b) $\dfrac{Tl_{cm}\cos(\theta+\phi)}{gL\cos(\phi)}$ c) $\dfrac{TL\cos(\theta+\phi)}{gl_{cm}\cos(\phi)}$ d) $\dfrac{TL\cos(\phi)}{gl_{cm}\cos(\theta)}$

e) none of the other answers provided

9) The expression for F_v is given by

a) $F_v = mg - T\sin(\theta)$
b) $F_v = -mg + T\cos(\theta)$
c) $F_v = mg + T\cos(\theta)$
d) $F_v = mg + T\sin(\theta)$
e) none of the other answers provided

Questions 10 - 12 pertain to a planet, p, having mass m revolving around a star, S, having mass M. Assume $M \gg m$. The planet's distance of closest approach to the star is r_0, and its farthest distance from the star is $\left(\frac{10}{9}\right) r_0$.



10) The ratio of the planet's speed v_1 at r_0 to its speed v_2 at $\left(\frac{10}{9}\right) r_0$ is given by

a) 100 to 81 b) 9 to 10 c) 10 to 9 d) 81 to 100 e) none of the other answers provided

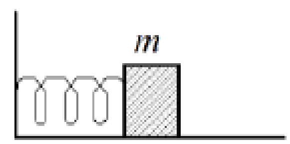
11) We know that that the planet's motion lies in a plane because

a) angular momentum is conserved b) kinetic energy is conserved c) total energy is conserved
d) linear momentum is conserved e) none of the other answers provided

12) When the planet is located at $\left(\frac{10}{9}\right) r_0$, the center of mass of the planet-star system is located at a distance from the star given by

a) $\frac{9}{10} r_0 \frac{m}{m+M}$ b) $\frac{9}{10} r_0 \frac{m+M}{m}$ c) $\frac{10}{9} r_0 \frac{m+M}{m}$ d) $\frac{10}{9} r_0 \frac{m}{m+M}$
e) none of the other answers provided

Questions 13 - 15 pertain to a mass m that is attached to a spring having spring constant k as shown. As shown, the spring is neither compressed nor stretched, so its relaxed length is L. The surface on which mass m rests has no friction. The spring is struck by a mallet and given a velocity v_0 toward the vertical axis.



13) After $\frac{3}{4}$ of a period, the mass m is located at a position along the horizontal axis given by

a) $L + v_0\sqrt{\frac{m}{k}}$ b) $L + 2v_0\sqrt{\frac{m}{k}}$ c) $L + v_0\sqrt{\frac{k}{m}}$ d) $L + \frac{v_0}{2}\sqrt{\frac{m}{k}}$

e) none of the other answers provided

14) The potential energy of mass-spring system when the spring is fully extended is given by

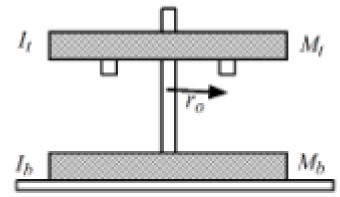
a) $\frac{1}{2}mv_0^2$ b) $2mv_0^2$ c) 0 d) mv_0^2 e) none of the other answers provided

15) When the speed of mass m is $\frac{v_0}{2}$, mass m is located at the position given by

a) $L \pm \sqrt{\frac{3k}{m}}\left(\frac{v_0}{2}\right)$ b) $L \pm \sqrt{\frac{3m}{k}}\left(\frac{v_0}{2}\right)$ c) $L \pm \sqrt{\frac{3m}{4k}}\left(\frac{v_0}{2}\right)$ d) $\sqrt{\frac{3m}{k}}\left(\frac{v_0}{2}\right) \pm L$

e) none of the other answers provided

Questions 16 - 18 pertain to the two-disk system shown. The bottom disk initially spins on a surface without friction with an angular velocity ω_0, whereas the top disk is initially at rest. The top disk drops down onto the bottom disk and makes contact at the rim a distance r_0 from its center. Friction between the rim of the top disk and the bottom disk causes slipping for a while, and eventually, the two disks spin as one. The masses and moments of inertial of the top and bottom disks, respectively, are shown in the figure. Let the coefficient of kinetic friction between the rim and the bottom disk be μ.



16) When slipping stops, the angular velocity ω_f of the system is given by

a) $\omega_f = \dfrac{I_t + I_b}{I_b}\omega_0$ b) $\omega_f = \dfrac{I_b}{I_t + I_b}\omega_0$ c) $\omega_f = \dfrac{I_t + I_b}{I_t}\omega_0$ d) $\omega_f = \dfrac{I_b}{I_t - I_b}\omega_0$

e) none of the other answers provided

17) The magnitude of the angular acceleration α of the top disk during the time it slips is given by

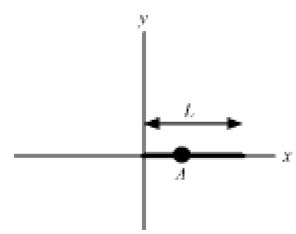
a) $\dfrac{r_0\mu M_t g}{I_t}$ b) $\dfrac{r_0 M_t g}{\mu I_t}$ c) $\dfrac{I_t}{r_0\mu M_t g}$ d) cannot be determined e) none of the other answers provided

18) The time required for the slipping to stop is given by

a) $\dfrac{I_b I_t \omega_0}{(I_t + I_b) r_0 \mu M_t g}$ b) $\dfrac{I_b I_t \omega_f}{(I_t + I_b) r_0 \mu M_t g}$ c) $\dfrac{I_b I_t \omega_0 r_0}{(I_t + I_b)\mu M_t g}$ d) $\dfrac{(I_t + I_b)\omega_0}{I_t I_b r_0 \mu M_t g}$

e) none of the other answers provided

Questions 19 - 21 pertain to a rod of length L that starts from rest and rotates about the origin at constant angular acceleration α_0 as shown. The rotation is counter-clockwise. Point A is located a distance $\dfrac{L}{3}$ from the origin.



19) The time required for the first one-half of a revolution of the rod is given by

a) $\sqrt{\dfrac{2\pi L}{\alpha_0}}$ b) $\sqrt{\dfrac{4\pi L}{\alpha_0}}$ c) $\sqrt{\dfrac{2\pi}{\alpha_0}}$ d) $\sqrt{\dfrac{4\pi}{\alpha_0}}$ e) none of the other answers provided

20) The vector expressions for the linear velocity $\mathbf{v}_A$ of point A and the angular acceleration $\boldsymbol{\alpha}$ of the rod when point A is as shown are given by

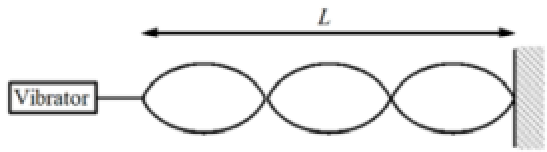
a) $\boldsymbol{\alpha} = -\alpha_0\hat{\mathbf{z}},\ \mathbf{v}_A = -v_A\hat{\mathbf{y}}$ b) $\boldsymbol{\alpha} = \alpha_0\hat{\mathbf{z}},\ \mathbf{v}_A = v_A\hat{\mathbf{x}}$ c) $\boldsymbol{\alpha} = \alpha_0\hat{\mathbf{z}},\ \mathbf{v}_A = v_A\hat{\mathbf{y}}$

d) $\boldsymbol{\alpha} = -\alpha_0\hat{\mathbf{z}},\ \mathbf{v}_A = v_A\hat{\mathbf{x}}$ e) none of the other answers provided

21) After a time $t = \sqrt{\dfrac{11\pi}{2\alpha_0}}$, the angle the rod makes with the positive x-axis, measured counter-clockwise, is given by

a) $\dfrac{\pi}{4}$ b) $\dfrac{5\pi}{4}$ c) $\dfrac{3\pi}{4}$ d) $\dfrac{7\pi}{4}$ e) none of the other answers provided

Questions 22 and 23 pertain to a string that is stretched between two supports. The tension in the string is T, the length of the string is L, and the speed of a wave on the string is v_0. The linear mass density of the string is d. One of the supports is a vibrator that can vibrate with frequency f causing a standing wave to be set up as shown.



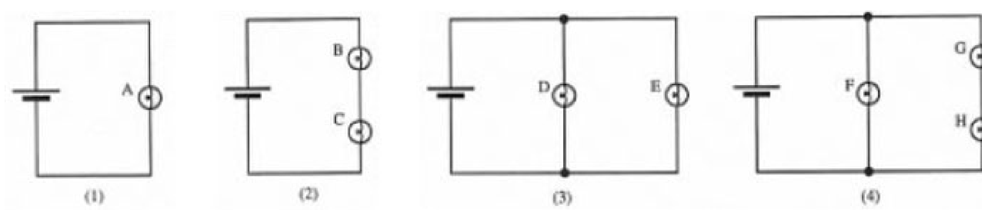
22) The frequency that causes this standing wave is given by

a) $\dfrac{2v_0}{3L}$ b) $\dfrac{v_0}{L}$ c) $\dfrac{v_0}{2L}$ d) $\dfrac{3v_0}{L}$ e) none of the other answers provided

23) If the linear mass density changes to $\dfrac{3}{4}d$, and no other parameters change, then in order to keep the same standing wave pattern, the frequency becomes

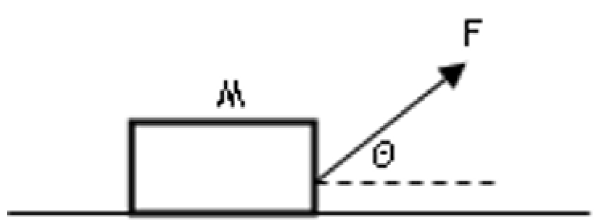
a) $\sqrt{\dfrac{3}{4}}\left(\dfrac{v_0}{L}\right)$ b) $\dfrac{3}{4}\left(\dfrac{v_0}{L}\right)$ c) $\sqrt{\dfrac{3}{2}}\left(\dfrac{v_0}{L}\right)$ d) $\sqrt{3}\left(\dfrac{v_0}{L}\right)$ e) none of the other answers provided

24) Rank the circuits according to the current through the battery from highest to lowest. Assume identical and ideal batteries, bulbs, and wires.



a) $2 > 1 > 4 > 3$ b) $1 > 2 > 3 > 4$ c) $4 > 3 > 2 > 1$ d) $3 > 4 > 1 > 2$
e) none of the other answers provided

25) In the diagram shown below, a force F is applied to a mass M at an angle θ to the horizontal. The mass is moving along a flat, rough, horizontal surface. What is the magnitude of the frictional force?



a) $\mu(Mg + F\sin(\theta))$ b) $\mu(Mg - F\sin(\theta))$ c) μMg d) $\dfrac{\mu Mg}{F\sin(\theta)}$ e) none of the other answers provided

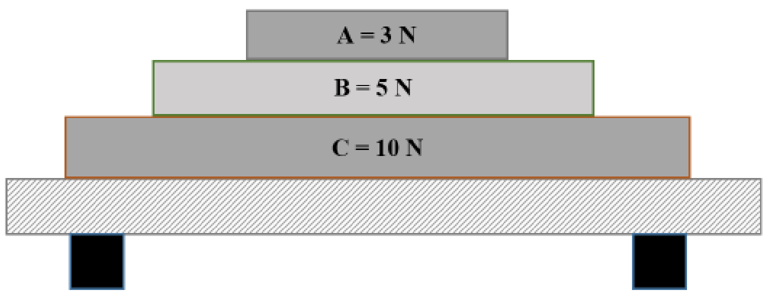
26) You are given three 2Ω resistors and many connecting wires. How many different total resistances could you create from these three, by using at least one of them at a time and connecting them in different ways? (Do not count 0Ω and $\infty\Omega$)

a) 8 b) 3 c) 7 d) 5 e) 6 f) none of the other answers provided

27) Juan and Luke travelled the same 600 km trip along a straight road. Juan travelled half the distance at 50 km/h and the other half at 100 km/h. Luke travelled for half his total travel time at 50 km/h and the remaining time at 100 km/h. Who arrived first, and by what time margin?

a) Luke arrived 1.0 h ahead of Juan. b) Both arrived at the same time. c) Juan arrived 1.0 h ahead of Luke.
d) Juan arrived 0.10 h ahead of Luke. e) Luke arrived 0.10 h ahead of Juan. f) none of the other answers provided

28) Three books A, B and C are piled on a table, as shown. The books and the table are at rest. The weight of each book is shown. What is the net force acting on book B?



a) 5 N [up] b) 0 N c) 2 N [down] d) 3 N [down] e) 8 N [down] f) none of the other answers provided

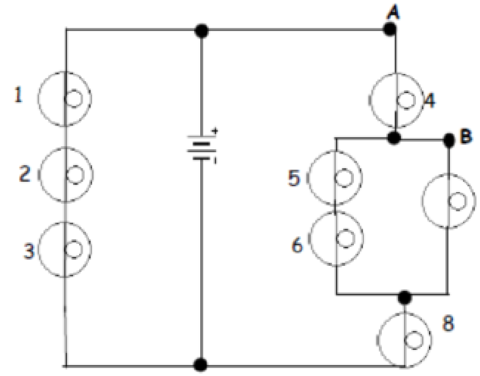
29) A wrench is dropped from rest off the top of the cliff with a height of h meters. It takes t seconds for the wrench to reach the bottom of the cliff. Where is the wrench at time $0.50t$?

a) The wrench is $0.25h$ m from the bottom of the cliff
b) The wrench is $0.75h$ m from the bottom of the cliff
c) The position depends on the mass of the wrench
d) The wrench is $0.50h$ m from the bottom of the cliff
e) none of the other answers provided

30) A mass is dropped from rest from the top of building of height h and strikes the sidewalk below at a speed of v. In order for the mass to strike with a speed of $2v$, the mass must be dropped from a height that is

a) three times higher b) two times higher c) four times higher d) 1.4 times higher
e) none of the other answers provided

Use the circuit at the right for questions 31 and 32.
All the light bulbs in the circuit have equal resistance.
The battery is ideal.
Lines represent ideal wires with zero resistance.



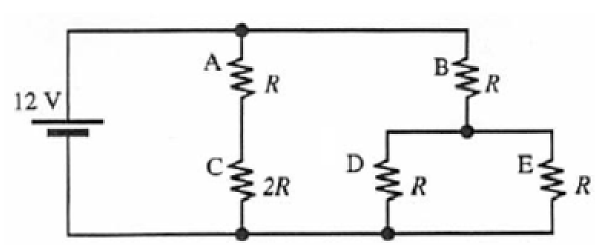
31) What is the ranking of the bulbs in order of decreasing brightness?

a) $1 = 2 = 3 > 7 > 4 = 8 > 5 = 6$
b) $1 = 2 = 3 > 4 = 8 > 5 = 6 = 7$
c) $4 = 8 > 7 > 5 = 6 > 1 = 2 = 3$
d) $4 = 8 > 1 = 2 = 3 > 7 > 5 = 6$
e) none of the other answers provided

32) Which of the following statements is true if we remove bulb 5 from its socket (and do nothing else to the circuit)?

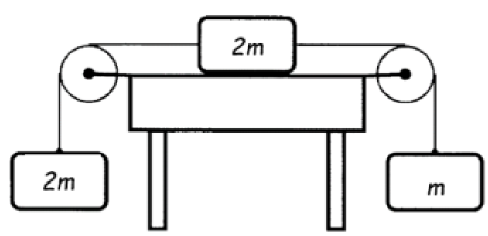
a) Bulb 1 will get brighter, bulb 4 will get dimmer, and bulb 7 will get dimmer
b) Bulb 1 will get dimmer, bulb 4 will get dimmer, and bulb 7 will get brighter
c) Bulb 1 will stay the same brightness, bulb 4 will get dimmer, and bulb 7 will get brighter
d) Bulb 1 will stay the same brightness, bulb 4 will get brighter, and bulb 7 will get dimmer
e) none of the other answers provided

33) Rank the resistors in the circuit shown according to the voltages across them from greatest to least.



a) $C = D = E > A = B$
b) $B = C > A = D = E$
c) $A = D = E > B = C$
d) $A = B > C = D = E$
e) none of the other answers provided

34) Three masses are connected with massless strings over frictionless pulleys as shown on a frictionless table. When they are released, what is the ratio of the tension in the string on the left to the string on the right? The masses are not drawn to scale.

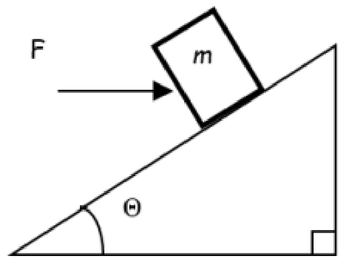


a) 1 : 2
b) 4 : 3
c) 3 : 4
d) 2 : 1
e) none of the other answers provided

35) Two small charged objects repel each other with a force F when separated by a distance r. If the charge on both of the objects is doubled and the distance between them is increased to $2r$, the force becomes:

a) $\frac{F}{16}$ b) $\frac{F}{4}$ c) $\frac{F}{2}$ d) F e) $\frac{F}{8}$ f) none of the other answers provided

36) A force of $F = 5\text{N}$ acts horizontally on a mass of $m = 1$ kg being pushed on a frictionless incline that makes an angle $\theta = 37^\circ$ with the horizontal. What is the magnitude of the acceleration of the mass?



a) $2\,\mathrm{m/s^2}$ b) $1\,\mathrm{m/s^2}$ c) $4\,\mathrm{m/s^2}$ d) $3\,\mathrm{m/s^2}$ e) none of the other answers provided

37) A 5.0-kg bucket is lowered by a rope in which there is 35 N of tension. What is the magnitude of the acceleration of the bucket?

a) $5.0\,\mathrm{m/s^2}$ b) $8.5\,\mathrm{m/s^2}$ c) $1.5\,\mathrm{m/s^2}$ d) $3.0\,\mathrm{m/s^2}$ e) none of the other answers provided

38) A man with a mass of 70 kg climbs at constant speed to the top of an 8 m vertical rope in 10 s. The average power expended by the man to overcome gravity is most nearly

a) 560 W b) 1.1 W c) 700 W d) 88 W e) none of the other answers provided

39) Two cannons are launched at the same time with the same speed from the same location. Both cannons land at the same spot on the ground. Cannon A is launched at an angle greater than 45° and Cannon B is launched at an angle less than 45°. Which cannon arrives first?

a) Both land at the same time b) Cannon A c) Cannon B
d) More information is needed e) none of the other answers provided

40) A student throws a baseball horizontally at a speed of 25 m/s from a cliff that is 45 m above the ground. How far from the base of the cliff does the ball hit the ground? Air resistance is negligible.

a) 75 m b) 230 m c) 140 m d) 45 m e) none of the other answers provided