

Precalculus Exam - University of Houston 2021 Math Contest
January 30, 2021

1) Given: $f(x) = 3e^{2x}$ and $g(x) = \log_5(4x + 1)$, evaluate $(f^{-1} \circ g^{-1})(2)$.

- a) $\ln(2)$ b) $\ln(\sqrt{2})$ c) $\ln(3)$ d) $\ln(4)$ e) $2\ln(2)$ f) $\ln\left(\frac{1}{2}\right)$ g) None of the above

2) Let $f(x^2 + 1) = x^4 + 3x^2 + 5$; find $f(-1) + f(-5) + 4f\left(-\frac{1}{2}\right) = ?$

- a) 42 b) 34 c) 16 d) 40 e) 21 f) 37 g) None of the above

3) Order the following numbers from smallest to greatest.

$$a = \sin\left(\frac{7\pi}{3}\right) + \cos\left(\frac{35\pi}{6}\right) \quad b = \sin\left(\frac{31\pi}{6}\right) + \cos\left(\frac{49\pi}{3}\right) \quad c = \sin\left(\frac{21\pi}{4}\right) + \cos\left(\frac{61\pi}{4}\right)$$

$$d = \sin\left(\frac{43\pi}{4}\right) + \sec\left(\frac{83\pi}{4}\right) \quad e = \tan\left(\frac{85\pi}{4}\right) + \cos\left(\frac{32\pi}{3}\right)$$

- a) $c < d < b < a < e$ b) $d < c < b < e < a$ c) $c < d < e < b < a$ d) $d < c < e < b < a$
e) $d < c < b < a < e$ f) $c < d < b < e < a$ g) None of the above

4) Given: $m - n = 3$ and $n - k = 5$, let $a = 5m^2 - 8n^2 + 3k^2$.

If $f(x) = 2x\cos(\pi x) + \sin\left(\frac{\pi}{2}x\right)$, find the value of $f(a)$.

- a) 480 b) 120 c) 600 d) 300 e) 360 f) 240 g) None of the above

5) Given $f(x) = 120 - 5\sin(\pi x + 1)$ and $g(x) = 20\tan(4x - 2)$; let P_1 and P_2 be the periods for the functions $f(x)$ and $g(x)$, respectively.

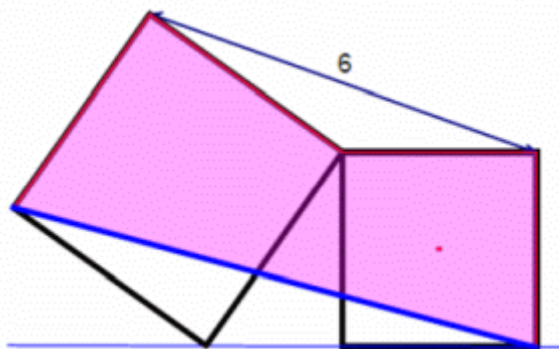
Find the value of: $\sin\left(\frac{P_2}{P_1}\right) \cos\left(\frac{P_2}{P_1}\right)$

- a) $\frac{\sqrt{6}}{4}$ b) $\frac{\sqrt{2}}{2}$ c) $\frac{\sqrt{2}}{4}$ d) $\frac{\sqrt{3}}{4}$ e) $\frac{\sqrt{6}}{2}$ f) $2\sqrt{2}$ g) None of the above

6) The following function is graphed over the interval $[0, \pi)$: $f(x) = 2\sin^2(4x) - 3\sin(4x) + 1$
Find the sum of the smallest and the largest x -intercepts of this function over this interval.

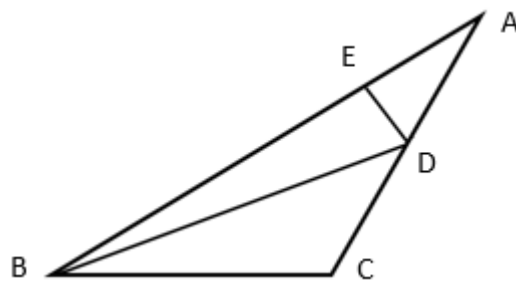
- a) $\frac{3\pi}{4}$ b) $\frac{11\pi}{24}$ c) $\frac{13\pi}{24}$ d) $\frac{3\pi}{8}$ e) $\frac{19\pi}{24}$ f) $\frac{\pi}{4}$ g) None of the above

38) There are two squares (non-identical) on the figure below. Find the area of the shaded region.
 (Note: The image may not be drawn to scale.)



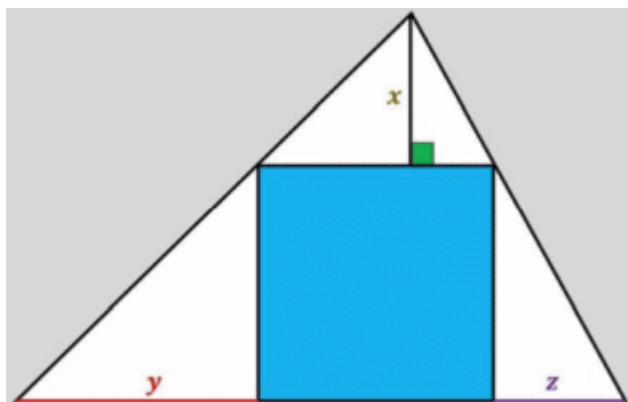
- a) 16 b) 18 c) 21 d) 20 e) 24 f) 27 g) None of the above

39) Given: $m(\angle EBD) = m(\angle DBC) = 15^\circ$, $m(\angle BED) = 90^\circ$, $ED = 2$, $BC = 6$. Find the area of the triangle ABC . (Note: The image may not be drawn to scale.)



- a) 27 b) 24 c) 16 d) 36 e) 20 f) 18 g) None of the above

40) The figure below contains a square inscribed in a triangle. Given: $x = 2$, $y = 3.1$, $z = 1.2$. Find the area of the square. (Note: The image may not be drawn to scale.)



- a) $\frac{41}{6}$ b) $\frac{43}{5}$ c) $\frac{33}{4}$ d) $\frac{79}{10}$ e) $\frac{81}{10}$ f) $\frac{38}{5}$ g) None of the above