Algebra 1 Exam - University of Houston Math Contest
January 29, 2022

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1) The top of one building is 16 feet higher than the top of another building. The heights of the two buildings are in the ratio $3{:}4$. In feet, how tall is the shorter building?

a) 48 b) 24 c) 32 d) 80 e) 96 f) 64

2) Using only pennies, nickels, dimes, and quarters, what is the smallest number of coins Jack would need so he could pay any amount of money less than a dollar?

a) 99 b) 10 c) 15 d) 25 e) 27 f) 6

3) For the picnic day, 50 students are wearing sunglasses and 35 students are wearing caps. Some students are wearing both sunglasses and caps. If one of the student wearing a cap is selected at random, the probability that this student is also wearing sunglasses is $\frac{2}{5}$. If instead, someone wearing sunglasses is selected at random, what is the probability that this student is also wearing a cap?

a) $\frac{14}{85}$ b) $\frac{4}{7}$ c) $\frac{7}{25}$ d) $\frac{2}{5}$ e) $\frac{7}{10}$ f) $\frac{7}{85}$

4) Anna has a bag of marbles. She gives 20% of them to her friend Marco. Then Anna gives 10% of what is left to another friend, Ella. Finally, Anna gives 25% of what is now left in the bag to her sister Emma. What percentage of her original bag of marbles does Anna have left for herself?

a) 38 b) 40 c) 45 d) $33\frac{1}{3}$ e) 54 f) 20

5) On a number line, what number is two-thirds of the way from $\frac{9}{4}$ back to $\frac{7}{8}$?

a) $\frac{37}{24}$ b) $\frac{43}{24}$ c) $\frac{4}{3}$ d) $\frac{11}{8}$ e) $\frac{13}{8}$ f) $\frac{11}{12}$

6) Consecutive integers are arranged in three columns in the pattern shown. What number will appear in column C in row 64?

A	B	C	
1	2	3	**Row 1**
6	5	4	**Row 2**
7	8	9	**Row 3**
12	11	10	**Row 4**
13	⋮	⋮	**Row 5**
⋮			

a) 184 b) 195 c) 172 d) 158 e) 189 f) 160

7) Sam has exactly $27.00 in dimes and quarters, with twice as many quarters as dimes. He spends five quarters and twice as many dimes at the convenience store, and he spends 55 cents at the donut shop. If he pays the exact amount of everything, how many quarters does Sam have left?

a) 54 b) 32 c) 39 d) 77 e) 84 f) 92

8) Kody's alarm rings every 15 minutes, and his dad calls upstairs every 11 minutes to wake him up. But Kody will only wake up if his alarm rings and his dad calls him at the same time. If his alarm first rings at 7:15am and his dad first calls him at 7:20am, at what time will Kody wake up?

a) 8:00am b) 8:15am c) 7:55am d) 8:25am e) 8:40am f) 8:50am

9) Two nomads were resting at an oasis when a third one, Yushin, showed up. Yushin asked them for some food since he had run out of it during his journey. The first nomad had 5 pieces of bread and the second one had 3. The three nomads shared the bread evenly. What is the ratio of the amount of bread given to Yushin by the first nomad to the amount of bread given by the second nomad?

a) 7:1 b) 1:1 c) 4:3 d) 1:7 e) 2:6 f) 6:2

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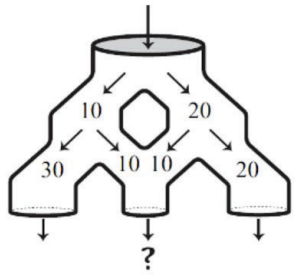
10) Six thousand marbles are released into the top pipe as shown and roll down the pipe system. Anytime the pipe forks, the marbles split in proportion to the cross-sectional areas of the pipes. All pipes have circular cross-sections with diameters as indicated in the figure below. How many marbles exit through the bottom middle pipe?



a) 2000 b) 1200 c) 1080 d) 1250 e) 960 f) 1500

11) Two of the solutions for x in the equation $x^3 - 9x = 5x^2 - 45$ are selected at random. These two values are the solutions for y in the equation $ay^2 + by + c = 0$ where a, b, c are integers with no common factors and $a > 0$. Determine the probability that $b < 0$.

a) $\frac{1}{3}$ b) $\frac{1}{5}$ c) $\frac{2}{3}$ d) $\frac{4}{5}$ e) $\frac{3}{4}$ f) $\frac{1}{2}$

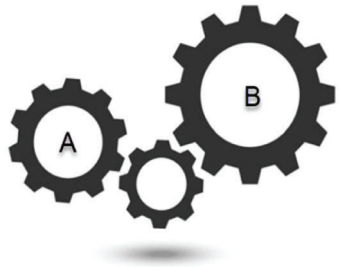
12) Paloma has a bag containing red, blue and white marbles. The ratio of red to blue marbles is 4:3, and the ratio of blue to white marbles is 7:2. What is the probability of Paloma randomly drawing a blue marble from this bag?

a) $\frac{4}{31}$ b) $\frac{21}{55}$ c) $\frac{28}{55}$ d) $\frac{4}{27}$ e) $\frac{21}{31}$ f) $\frac{7}{9}$

13) If $x^2 - 5x + 2 = 0$, find the value of $x^3 + \frac{8}{x^3}$.

a) 95 b) 45 c) 115 d) 105 e) 75 f) 100

14) In the figure shown below, the cogwheel labeled A makes one revolution every 2 seconds. How many revolutions will cogwheel B make every minute?



a) 36 b) 10 c) 12 d) 25 e) 20 f) 16

15) At Memorial High School, students are enrolled in exactly two of the school's after-school clubs or none of them. One-half of the students are in the Science Club, one-half of the students are in the Math Club, and one-half of the students are in the Music Club. If you were to pick a student at random from this school, what is the probablity that they are not involved at any clubs?

a) $\frac{3}{8}$ b) $\frac{1}{2}$ c) $\frac{1}{4}$ d) $\frac{2}{5}$ e) $\frac{1}{3}$ f) 0

16) If $10^{2022} - 2022$ is written as an integer in its decimal form, what is the sum of its digits?

a) $18,202$ b) $18,193$ c) $20,184$ d) $18,229$ e) $18,162$ f) $18,189$

17) Given the following sequence of numbers:

$$98, 79, X, 47, 34, 23, Y, 7, 2, Z$$

Calculate the value of $100X + Y - Z$.

a) 6213 b) 5814 c) 6214 d) 5813 e) 6215 f) 6114

18) Let $w, x, y,$ and z be whole numbers. If $2^w \cdot 3^x \cdot 5^y \cdot 7^z = 882$, then what does $2w + 3x + 5y + 7z$ equal?

a) 46 b) 25 c) 22 d) 35 e) 60 f) 21

19) If 3 different integers are randomly chosen from the 20 smallest positive integers, what is the probability that their product is a multiple of 5?

a) $\frac{25}{27}$ b) $\frac{31}{57}$ c) $\frac{29}{57}$ d) $\frac{28}{57}$ e) $\frac{17}{19}$ f) $\frac{26}{57}$

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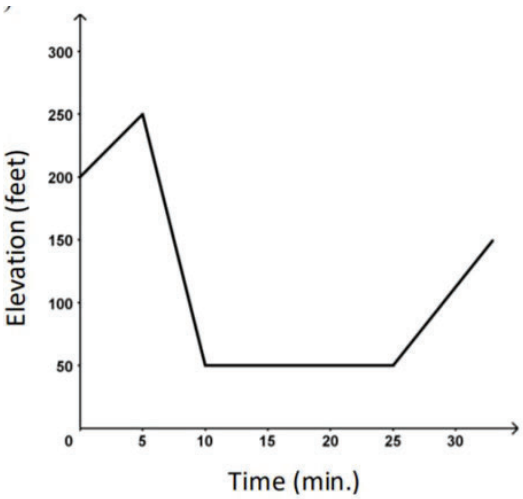
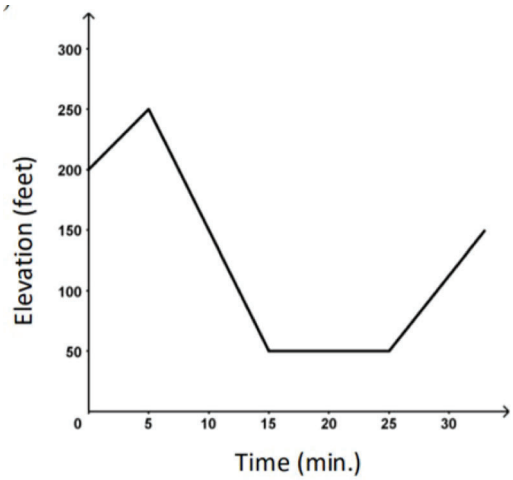
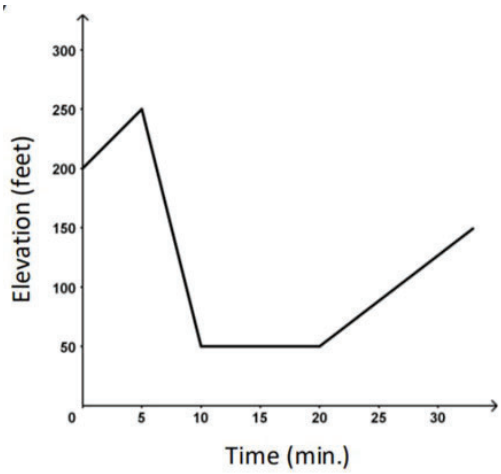
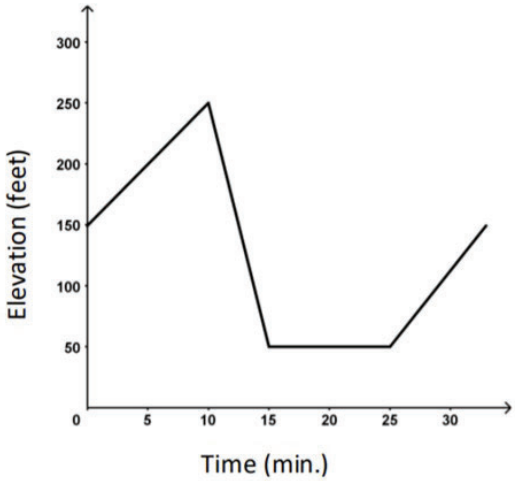
20) Mark started out by walking up a hill for 5 minutes. For the next 5 minutes, he walked down a steep hill to an elevation lower than his starting point. For the next 10 minutes, he walked on level ground. For the next 10 minutes, he walked uphill. Determine which graph of elevation above sea level versus time illustrates the story.

a)


b)


c)


d)


e) None of the above.

21) In a family with 9 siblings, the average age of the 5 youngest siblings is 6 and the average age of the 5 oldest siblings is 18. What is the difference between the average age of the 4 oldest siblings and the average age of the 4 youngest ones?

a) 15 b) 14 c) 10 d) 16 e) 13 f) 12

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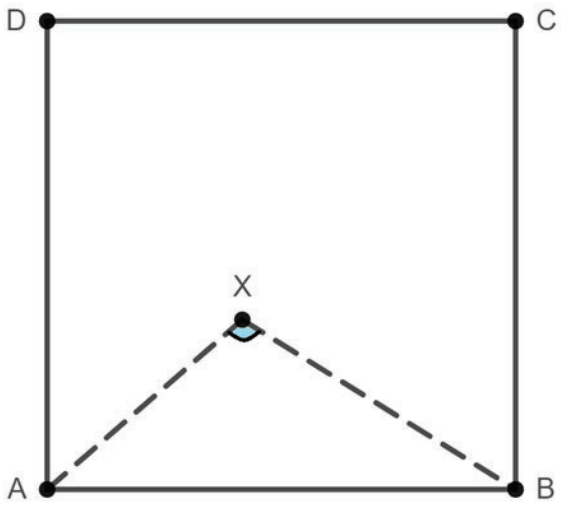
22) Point X is randomly selected within square $ABCD$. What is the probability that the angle AXB is obtuse?



a) $\frac{1}{4}$ b) $\frac{8-\pi}{8}$ c) $\frac{\pi}{8}$ d) $\frac{\pi}{4}$ e) $\frac{4-\pi}{4}$ f) $\frac{1}{8}$

23) There are ten socks in a drawer. Two are white, two are black, two are red, and the rest are gray, green, yellow and blue. If you pick 2 socks out of the drawer at random, what is the probability that you choose a match?

a) $\frac{2}{45}$ b) $\frac{1}{15}$ c) $\frac{2}{5}$ d) $\frac{2}{15}$ e) $\frac{1}{90}$ f) $\frac{3}{5}$

24) A group of students were planning a field trip. Each day, they wanted to share 1 bottle of water among each 4 students for breakfast, one between each 2 for lunch, and another one among each 3 for dinner. Assume that each bottle is fully consumed and shared equally at each meal. What is the minimum number of water bottles they needed per day?

a) 13 b) 24 c) 26 d) 12 e) 14 f) 9

25) Noah, Emma, and Oliver have an algebra homework due tonight. If they did it individually, Noah would finish in 24 minutes, Emma in half an hour, and Oliver in 40 minutes. To save some time, they decide to do it together. Assume that they are perfectly efficient in dividing the tasks among them. How long will it take them to do the homework?

a) 20min b) 10min c) 12min d) 27min e) 15min f) 8min

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26) Suppose there is an 80% chance of rain each day. On days that it rains, Kathy has a 45% chance of being late, compared to a 30% chance when it does not rain. On a random day, what is the probability that Kathy will be late?

a) 36% b) 75% c) 42% d) 9% e) 55% f) 15%

27) Ella started saving money in her empty piggy bank. She inserted $\$1$ on every Monday, $\$2$ on every Tuesday, $\$3$ on every Wednesday, $\$4$ on every Thursday, $\$5$ on every Friday, $\$6$ on every Saturday, and $\$7$ on every Sunday. At the end of 31 days, she opened her bank and found she had $\$127$. On which day of the week did she open her piggy bank?

a) Thursday b) Saturday c) Sunday d) Monday

e) Wednesday f) Friday g) Tuesday

28) In David's sports league, each team plays at most one game per day. Furthermore, no team is allowed to play games on three consecutive days, nor may any team play four or more games in any five consecutive days. Under these constraints, what is the maximum number of games David's team could play in a 108-day interval?

a) 55 b) 66 c) 23 d) 24 e) 65 f) 36

29) Jill is packing marbles into boxes. She needs to pack away 815 marbles. She has boxes that can hold $10, 25, 50$ or 100 marbles. If Julie can use at most 5 boxes of each size and must fill each box she uses, what is the minimum number of boxes she needs to pack all her marbles?

a) 17 b) 15 c) 28 d) 21 e) 30 f) 25

30) In Mr. Patterson's class, the average score among students who studied for an exam was 82. The average among students who did not study was 50. The overall class average was 74. What portion of the class did not study?

a) $\frac{1}{4}$ b) $\frac{2}{3}$ c) $\frac{3}{4}$ d) $\frac{2}{5}$ e) $\frac{3}{5}$ f) $\frac{1}{3}$

31) Suppose that $p(x)$ is a function such that $p(x) = p(1) + p(2) \cdot x + x^2$ for all real numbers x. What is the value of $p(7)$?

a) 29 b) 39 c) 59 d) 47 e) 51 f) 42

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32) Some numbers are hidden in this sequence of positive integers, which contains each integer from 1 to 13 exactly once. Every term after the first term is 1 more than the previous term, 2 more than the previous term, or 3 less than the previous term. What is the product of the two numbers represented by A and B?

1, [redacted], A, 6, [redacted], [redacted], 7, B, [redacted], 12, [redacted], [redacted], 13

a) 50 b) 99 c) 32 d) 33 e) 40 f) 80

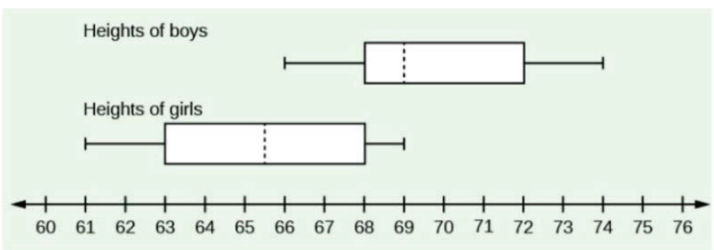
33) If $x + y = 5$ and $x^2 + y^2 = 97$, then find $(|x| + |y|)^3$.

a) 2197 b) 125 c) 485 d) 1279 e) 1297 f) 2179

34) If x and y are positive real numbers such that $x + y = 3$, which of the following could be the value of $30x + 40y$?

a) 45 b) 70 c) 135 d) 160 e) 95 f) 87

35) The following box plots represent the heights of the boys and girls of a classroom:



With the information provided by the diagram, which of the following statements can we certainly say is true about the sample?

a) The shortest boy is taller than the tallest girl.
b) The range of the heights in the class is one foot and one inch.
c) The class consists of 8 girls and 8 boys.
d) The tallest girl is as tall as the mean boy.
e) We cannot determine any of the above statements as true.

36) The cost $C(x)$ for manufacturing x units of a certain product is given by $C(x) = x^2 - 10x + 600$, and the units sell for $95 each. Find the number of units manufactured at a cost of $7800 and the profit or loss from the manufacture and sale of those units.

a) 80 units; loss of $200 b) 90 units; loss of $450 c) 70 units; loss of $1150

d) 70 units; profit of $4950 e) 90 units; profit of $750 f) 80 units; profit of $7600

37) When the polynomial $p(x) = x^3 - 3x^2 + ax + b$ is divided by $x - 1$, the remainder is -4, and when $p(x)$ is divided by $x - 2$ the remainder is also -4. Find the remainder when $p(x)$ is divided by $x - 3$.

a) -4 b) 0 c) -2 d) 1 e) 2 f) -1

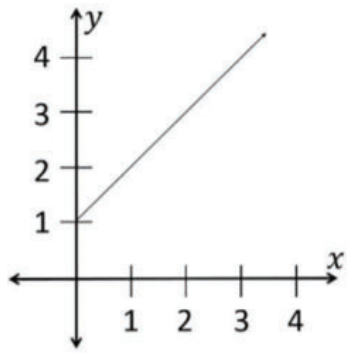
38) The following functions $f,\ g,$ and h represent the distance y over time x travelled by three different objects (measured in the same units):

f:

x	y
0	1
5	6
10	11

g: $x = \frac{y}{4} - \frac{3}{8}$

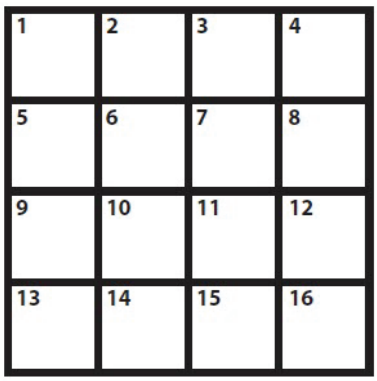
h:



Which object is moving faster?

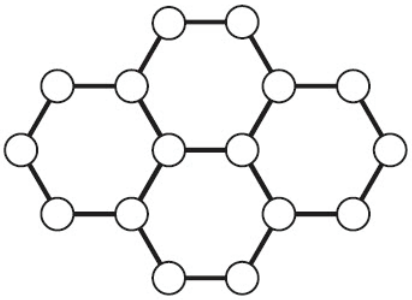
a) All the objects are moving at the same speed.
b) The answer cannot be determined with the information provided
c) The objects represented by function f and h, which are travelling at the same speed.
e) The object represented by function h
f) The object represented by function f

39) How many ways are there to color the 4×4 grid shown, so that each unit square is red, blue, green or yellow, and so that each unit square is the same color as exactly two of the unit squares that share a side with it?



a) 96 b) 108 c) 90 d) 84 e) 60 f) 72

40) **TIEBREAKER QUESTION:** The integers from 1 to 16 are placed in the 16 circles in the figure shown below, with each number occurring exactly once and so that the sum of the six numbers around each hexagon is S. What is the least possible value of S?



a) 62 b) 80 c) 55 d) 42 e) 40 f) 21