

## Calculator Exam - University of Houston Math Contest January 29, 2022

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### EXAM INSTRUCTIONS:

Some problems are easier than others. There is no penalty for skipping problems or guessing. Unless otherwise requested, select an answer that is closest to the exact answer, unless there is no answer within  $10^{-4}$  of the exact answer. If no such value appears, select 'None of the above.'

1) Give the largest value of  $x$  where the graph of

$$f(x) = \frac{1}{4}x^3 - 3x^2 - x + 8$$

crosses the  $x$ -axis.

a) 12.1120      b) 12.1125      c) 12.1121      d) 12.1124      e) 12.1123      f) None of the above.

2) Use the function  $f(x)$  defined in problem 1. Give the average of the smallest and largest roots of  $f(x)$ .

a) 5.2146      b) 5.2148      c) 5.2145      d) 5.2147      e) 5.2149      f) None of the above.

3) Give the sum of the reciprocals of the first 100 positive integers.

a) 5.1871      b) 5.1870      c) 5.1873      d) 5.1869      e) 5.1872      f) None of the above.

4) Find the  $x$ -coordinate of the point on the graph of  $y = 2x - 3$  that is closest to the point  $(-1, 4)$ .

a) 2.9      b) 2.7      c) 2.5      d) 3.0      e) 2.8      f) None of the above.

5) Give the largest slope of a line between two distinct points chosen from the list:  
 $(\cos(1), \sin(1)), (\cos(2), \sin(2)), \dots, (\cos(20), \sin(20))$ .

Note: All angle values are given in radians.

a) 15.09736      b) 15.06332      c) 15.05443      d) 15.04637  
e) 15.04478      f) None of the above.

6) Give the obtuse angle of intersection (in radians) of the lines  $47x + 21y = -12$  and  $213x - 18y = 17$ .

- a) 2.91919    b) 2.78132    c) 2.64306    d) 2.63709    e) 2.57214    f) None of the above.

7) Give the largest value of  $c$  so that if  $a > c$  then the graph of  $y = x^2 - 2x + a$  lies above the line  $y = -3x + 2$ .

- a) 2.26    b) 2.22    c) 2.24    d) 2.25    e) 2.21    f) None of the above.

8) Define the function

$$f(x) = \frac{1}{6}x^2 - \frac{1}{4}x + 1.$$

Find  $f(f(f(f(1))))$ .

- a) 0.91013    b) 0.91071    c) 0.91054    d) 0.91032    e) 0.91068    f) None of the above.

9) Define the function

$$f(x) = -1 + \frac{1}{1^2 + x} + \frac{1}{2^2 + x} + \frac{1}{3^2 + x} + \dots + \frac{1}{10^2 + x}.$$

Let  $x_0 = 0.1$  and define  $x_n = f(x_{n-1})$  for  $n = 1, 2, \dots$ . The values of  $x_n$  get closer and closer to a value of  $x$  as  $n$  gets larger. Give this  $x$  value.

- a) 0.2983    b) 0.2975    c) 0.2972    d) 0.2977    e) 0.2969    f) None of the above.

10) Janis invested money in a mutual fund in 2011. During the next 10 years, the annual rates of return were

7.1%, 6.9%, 8.3%, 9.7%, 6.4%, 4.2%, 11.1%, 9.1%, -2.4%, 1.3%

What was the average annual rate of return?

- a) 6.17%    b) 6.33%    c) 6.13%    d) 6.22%    e) 6.07%    f) None of the above.

11) Give the value of  $y$  associated with the solution to the system

$$\begin{cases} 47x + 21y = -1200 \\ 213x - 18y = 1700 \end{cases}$$

- a) -63.04312    b) -63.04417    c) -63.05611    d) -63.06142

- e) -63.07576    f) None of the above.

12) A36 carbon steel plate has a density of 0.2836 pounds per cubic inch. The table below shows the length (inches), width (inches), thickness (inches), and number of steel plates of various sizes that are scheduled for shipment to a fabrication site. Give the total weight of the steel plates (to the nearest pound) that will be sent to the fabrication site.

Number	Length	Width	Thickness
10	360	96	0.5
20	240	48	0.75
11	480	120	0.325

- a) 156,523 pounds      b) 156,617 pounds      c) 156,726  
d) 156,322 pounds      e) 156,411 pounds      f) None of the above.

13) Circles with radius 0.12 are placed at each point  $(i, j)$  for  $i = 1, \dots, 10$  and  $j = 1, \dots, 10$ . Give the number of circles intersected by the line  $y = 3x - 0.48$ .

- a) 0      b) 4      c) 1      d) 2      e) 3      f) None of the above.

14) Give the area of the triangle that passes through the points  $(-12, 13)$ ,  $(13, 2)$ , and the vertex of the parabola  $y = -3x^2 - 17x + 22$ .

- a) 463.9411      b) 463.9813      c) 463.9647      d) 463.9672  
e) 463.9583      f) None of the above.

15) A point  $(x, y)$  is called an integer point if both  $x$  and  $y$  are integers. Give the number of integer points that lie strictly above the parabola  $y = \frac{1}{2}x^2 - 3x - 2$  and strictly below the parabola  $y = -\frac{1}{10}x^2 + 6x + 163$ .

- a) 4813      b) 4221      c) 4632      d) 4086      e) 4473      f) None of the above.

16) A game is created where each player gets ten turns spinning a dial that selects numbers 1 through 12. The numbers 1 through 12 have an equal chance of being selected when the dial is spun, and the points the players receive when a number is spun are shown in the table below. A player's total score for the game is the sum of the scores on the ten spins of the dial. What is the average number of points that players will receive for playing this game?

number	1	2	3	4	5	6	7	8	9	10	11	12
points	340	-200	140	-300	420	-500	82	120	-300	4	-200	400

- a) 84      b) 102      c) 20      d) 35      e) 16      f) None of the above.

17) Give the  $y$ -coordinate of the  $y$ -intercept of the line that passes through the point  $(21, 3)$  and is perpendicular to the line that passes through the points  $(4, 12)$  and  $(11, -215)$ .

- a) 2.3864      b) 2.3793      c) 2.3271      d) 2.3421      e) 2.3524      f) None of the above.

18) A sandwich shop serves over 1000 customers each day, seven days per week, and the management decides to boost interest in its sandwiches by giving a free sandwich to a select number of customers each day. The number of customers who will receive a free sandwich on a given day is determined by a drawing that the manager and assistant manager have each morning from a sack that has 5 tokens with the numbers 5, 12, 15, 17 and 20. The number on the token that is drawn determines the number of customers who will each receive one free sandwich that day. Suppose there is an equal likelihood of selecting each token. How many positive integers from 1 to 140 represent numbers of sandwiches that will never be the total number of sandwiches given away free in a week?

- a) 46      b) 67      c) 51      d) 63      e) 58      f) None of the above.

19) If the number 819 is written in base 2, then how many digits in the base 2 expression are 1?

- a) 7      b) 5      c) 4      d) 6      e) 8      f) None of the above.

20) A number is written in base 3 as 1202121021. Give this number in base 10.

- a) 34,702      b) 34,661      c) 34,552      d) 34,824      e) 34,937      f) None of the above.

21) Give the number of solutions to the equation

$$\frac{1}{4}x^3 - 3x^2 + 8 = x - 20 \sin(15x).$$

- a) 35      b) 33      c) 29      d) 37      e) 31      f) None of the above.

22) A point  $(x, y)$  is called an integer point if both  $x$  and  $y$  are integers. Suppose a triangle is created in the  $xy$ -plane with vertices at integer points. Of the values given below, which one could represent the area of a triangle whose vertices are integer points?

- a) 17.25      b) 14.75      c) 13.5      d) 15.60      e) 6.125      f) None of the above.

23) A particle travels along a path in the  $xy$ -plane, starting from  $(1, 2)$ , moving along a line segment to the point  $(13, 17)$ , making one revolution of the circle of radius 2 centered at  $(13, 19)$ , and then returning along the line segment to  $(1, 2)$ . How many units did the particle travel?

- a) 50.7634      b) 50.9851      c) 50.8632      d) 51.2113      e) 50.6536      f) None of the above.

24) What number is represented by the curve given parametrically by  $x = \sin(2t)$ ,  $y = \cos(t)$ , for  $0 \leq t \leq 2\pi$ ?

- a) 6      b) 3      c) 8      d) 1      e) 0      f) None of the above.

25) Find the product

$$\left(\frac{1}{2} + \frac{1}{4}\right) \times \left(\frac{2}{3} + \frac{2}{9}\right) \times \left(\frac{3}{4} + \frac{3}{16}\right) \times \dots \times \left(\frac{100}{101} + \frac{100}{101^2}\right)$$

- a) 0.5346      b) 0.5163      c) 0.5049      d) 0.5232      e) 0.4997      f) None of the above.

26) Let  $C$  be the circle of radius 1 centered at the origin. 32 equally spaced points are placed on  $C$ , with the first point  $P_1 = (1, 0)$ , and the other 31 points  $P_2, \dots, P_{32}$  ordered counterclockwise around the circle. Give the sum of the absolute values of the  $x$  and  $y$  coordinates for these 32 points.

- a) 40.6489      b) 40.5933      c) 40.6072      d) 40.6127      e) 40.6324      f) None of the above.

27) **Tie Breaker:** Give the sum of the solutions to the equation

$$\frac{1}{4}x^3 - 3x^2 + 8 = x - 20 \sin(15x).$$

- a) 63.8883  
b) 63.9283  
c) 63.9183  
d) 63.8983  
e) 63.8583  
f) 63.8483  
g) 63.9083  
h) 63.8783  
i) 63.9683  
j) 63.8383  
k) 63.8683  
l) 63.9383  
m) 63.9483  
n) 63.9583  
o) 63.9783  
p) None of the above.