

Precalculus Exam - University of Houston Math Contest
January 29, 2022

1) What is the distance between vertices of the parabolas $f(x) = 4 - x^2$ and $g(x) = (x + 3)^2$?

- a) 2 b) 7 c) 5 d) 4 e) none of the other answer choices provided

2) If the graph of $f(x) = (m - 10)x^3 + x^{n-6} + 2x + 1$ is a parabola then what is the value of $m + n$?

- a) 10 b) 18 c) 20 d) 12 e) none of the other answer choices provided

3) Line L contains the points $(2, 1)$ and $(1, -2)$, and Line M contains the points $(-2, 9)$ and $(10, -3)$. What is the y-coordinate of the point of intersection of lines L and M?

- a) 4 b) -1 c) 3 d) 7 e) none of the other answer choices provided

4) If $y = 6x - 1$ and $y = ax + 4$ are the equations of perpendicular lines, then what is the value of a ?

- a) $\frac{1}{6}$ b) -6 c) $-\frac{1}{6}$ d) 6 e) none of the other answer choices provided

5) If $f^{-1}(x) = \sin^{-1}\left(\frac{x}{2}\right)$ and $g^{-1}(x) = \cot^{-1}\left(\frac{-3}{2-x}\right)$ then what is the value of $(f^{-1} \circ g)\left(\frac{3\pi}{4}\right)$?

- a) $-\frac{\pi}{3}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $-\frac{\pi}{6}$ e) $\frac{\pi}{6}$

f) none of the other answer choices provided

6) Evaluate the following trigonometric expression.

$$\frac{\sin(x) + \sin(3x) + \sin(7x) + \sin(9x)}{\sin(5x) \cos(3x) \cos(x)}$$

- a) 3 b) 1 c) 4 d) 2 e) 0 f) none of the other answer choices provided

7) If $\frac{\pi}{2} < x < \pi$ and $\sin\left(\frac{3\pi}{2} + x\right) = \frac{3}{5}$ then the value of $\sin(x)$ is:

- a) $\frac{4}{5}$ b) $\frac{3}{5}$ c) $\frac{2}{5}$ d) $\frac{1}{5}$ e) none of the other answer choices provided

8) A phone company charges k dollars for the first hour of use in a month and m dollars per hour for every additional hour used that month. If Ali paid 65.50 dollars for his phone use in one month, which of the following expressions represents the number of hours he used the phone that month?

- a) $\frac{65.50 - k}{m}$ b) $\frac{65.50}{k + m}$ c) $\frac{65.50 - k - m}{m}$ d) $\frac{65.50 - k + m}{m}$

e) none of the other answer choices provided

9) If the parabola $y = nx^2 - (3n - 5)x + n + 2$ intersects the y-axis at the point $(0, -3)$, which of the following is the x-coordinate of the vertex of the parabola?

- a) 1 b) 2 c) 7 d) 0 e) none of the other answer choices provided

10) Find the exact value of $\sin\left(\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{3}{5}\right)\right)$.

- a) $\frac{4\sqrt{3} + 3}{10}$ b) $\frac{4\sqrt{3} - 3}{10}$ c) $\frac{3\sqrt{4} + 3}{10}$ d) $\frac{3\sqrt{4} - 3}{10}$

e) none of the other answer choices provided

11) Given that $\cos(8^\circ) = a$, evaluate $\sin(98^\circ) - \cos(188^\circ) + 2\sin(278^\circ)$

- a) $a + 2$ b) $2a$ c) 0 d) 1 e) none of the other answer choices provided

12) Simplify the following expression

$$\frac{\tan^2(x)}{\sec^2(x)} + \frac{\cot^2(x)}{\csc^2(x)}$$

- a) $2\sin(x)$ b) 1 c) $\tan(x)$ d) $2\cos(x)$

e) none of the other answer choices provided

13) Given: $\tan(\pi - x) = \frac{2}{3}$, evaluate the value of

$$\frac{\sin x \cos x}{\cot\left(\frac{3\pi}{2} - x\right)}$$

- a) $\frac{3}{13}$ b) $\frac{7}{13}$ c) $\frac{1}{9}$ d) $\frac{9}{13}$ e) none of the other answer choices provided

14) Which of the following is the period of the function $f(x) = 2 - 3 \sin^6\left(\frac{x-1}{5}\right)$.

- a) 2π b) 4π c) 6π d) 5π e) 3π
f) none of the other answer choices provided

15) Consider the equation $4 \arctan(x^2 + x - 13) = -\pi$: Find the sum of the solutions of the equation.

- a) -1 b) 1 c) 2 d) 0 e) none of the other answer choices provided

16) Consider $\triangle ABC$ with $\angle CBA = 135^\circ$, $|BC| = 2\sqrt{2}$ in, $|AB| = 3$ in. Find the area of the triangle.

- a) 6 in^2 b) 3 in^2 c) 27 in^2 d) 9 cm^2
e) none of the other answer choices provided

17) Find the smallest solution for the equation $\tan(2x) + \cot(2x) = 2\sqrt{2}$ on $\left[0, \frac{\pi}{2}\right)$.

- a) $\frac{\pi}{16}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{8}$ d) $\frac{\pi}{4}$ e) none of the other answer choices provided

18) Find the domain of the function $f(x) = \sqrt[4]{2 - \log(x-2)}$.

- a) $[3, \infty)$ b) $(2, 102]$ c) $(-\infty, 3]$ d) $(2, \infty)$
e) none of the other answer choices provided

19) Find the radius of the circle $x^2 + y^2 - 8x + 2y = -8$.

- a) 3 b) 8 c) 4 d) 6 e) none of the other answer choices provided

20) Kris, Alex, and John are playing basketball. Kris passes the ball to Alex when Kris is 12 feet from John and 10 feet from Alex. How far is Alex from John if the angle from John to Kris to Alex (i.e. with Kris at the vertex) is 60 degrees?

- a) $2\sqrt{31}$ feet b) $4\sqrt{41}$ feet c) $4\sqrt{91}$ feet d) $2\sqrt{21}$ feet e) $4\sqrt{35}$ feet
f) none of the other answer choices provided

21) If $\vec{a} = \langle 1, -2 \rangle$ and $\vec{b} = \langle 5, -1 \rangle$, what is the magnitude of the vector $2\vec{a} - \vec{b}$?

- a) $2\sqrt{3}$ b) $3\sqrt{2}$ c) $\sqrt{2}$ d) $2\sqrt{2}$ e) none of the other answer choices provided

22) Find the number of points where the circle $x^2 + (y - 4)^2 = 25$ and the parabola $y = -x^2 + 9$ intersect.

- a) 3 b) 0 c) 4 d) 1 e) none of the other answer choices provided

23) How many zeros does $f(x) = 2x^5 - 7x^4 + 9x^3 - 18x^2 + 4x + 40$ (Including multiplicities)?

- a) 3 b) 2 c) 4 d) 5 e) none of the other answer choices provided

24) Solve: $4^x - 2^x - 12 = 0$

- a) 3 b) 2 c) 1 d) 7 e) none of the other answer choices provided

25) How many solutions does $4\sin^2(\theta) - 3 = 0$ have on $0 \leq \theta < 2\pi$?

- a) 4 b) 0 c) 2 d) 3 e) none of the other answer choices provided

26) Identify the graph of the equation $6x^2 + 3y^2 - 12x + 6y = 0$.

- a) hyperbola b) parabola c) circle d) ellipse
e) none of the other answer choices provided

27) Simplify the following expression.

$$\frac{\sec^2(v) - \tan^2(v) + \tan(v)}{\sec(v)}$$

- a) $\tan(v)$ b) $\sin(v) - 1$ c) $\sin(v) + \cos(v)$ d) $\cos(v) - \sin(v)$
e) none of the other answer choices provided

28) Which of the following is **not** a solution of the equation $\tan(2\theta) + 2\cos(\theta) = 0$ on the interval $0 \leq \theta < 2\pi$.

- a) $\frac{7\pi}{6}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{6}$ d) $\frac{3\pi}{2}$ e) none of the other answer choices provided

29) The vector $\vec{v} = i + \sqrt{3}j$ find the direction angle of \vec{v} .

- a) 60° b) 30° c) 45° d) 90° e) none of the other answer choices provided

30) In a Laser projections system, the optical and scanning angle is related to the throw distance D from the scanner to the screen and the projected image width W by the equation

$$D = \frac{\frac{1}{2}W}{\csc(\theta) - \cot(\theta)}$$

Which of the following is the projected image width?

- a) $W = 2D \tan\left(\frac{\theta}{2}\right)$ b) $W = 2D \tan(\theta)$ c) $W = 4D \tan\left(\frac{\theta}{4}\right)$
d) $W = D \tan\left(\frac{\theta}{2}\right)$ e) none of the other answer choices provided

31) Given that $\sqrt{x} = \sqrt{y} - \frac{1}{\sqrt{y}}$, write the following expression in terms of y :

$$\frac{x + 2 - \sqrt{x^2 + 4x}}{x + 2 + \sqrt{x^2 + 4x}}$$

- a) y^2 b) y^{-2} c) $2y^{-1}$ d) $2y$ e) $1 + y^{-2}$
f) none of the other answer choices provided

32) Given that $\tan^3(x) + \cot^3(x) = 18$, find the value of $\tan^7(x) + \cot^7(x) + 7$.

- a) 850 b) 820 c) 920 d) 1220 e) 950
f) none of the other answer choices provided

33) **TIEBREAKER QUESTION.** Given: $\tan(x) + \cot(y) = 1$ and $\tan^2(x) + \cot^2(y) = 2$, find the value of $8 \tan^8(x) + 8 \cot^8(y)$.

- a) 97 b) 108 c) 174 d) 49 e) 79
f) none of the other answer choices provided