

**Algebra 1 Exam - University of Houston 2023 Math Contest**  
**January 28, 2023**

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1) Find the distance between the following points:  $(2\sqrt{3}, 5\sqrt{6})$  and  $(-\sqrt{3}, \sqrt{6})$ .

- a)  $\sqrt{123}$       b)  $\sqrt{33}$       c)  $\sqrt{63}$       d) 11      e) 21      f) None of these

2) Write an equation of the line that passes through the point  $(1, 2)$  and is perpendicular to the line  $y = -2x + 4$ .

- a)  $y = \frac{1}{2}x$       b)  $y = \frac{1}{2}x + \frac{3}{2}$       c)  $y = -\frac{1}{2}x + 3$       d)  $y = -\frac{1}{2}x - 3$       e)  $y = -\frac{1}{2}x - \frac{3}{2}$

f) None of these

3) Solve the equation for the variable  $x$ :  $-\frac{3}{8x} + \frac{1}{12x} = 2$ .

- a) There is no solution.      b)  $x = -\frac{5}{24}$       c)  $x = \frac{11}{48}$       d)  $x = -\frac{7}{48}$       e)  $x = -\frac{7}{24}$       f) None of these

4) If the first and third of three consecutive odd integers are added, the result is 63 less than five times the second integer. What is the product of the first and third integer?

- a) 117      b) 437      c) 364      d) 357      e) 77      f) None of these

5) Find all real solutions of  $6x^2 - 6x = -1$

- a)  $\frac{1}{4} \pm \frac{\sqrt{15}}{4}$       b)  $\frac{1}{3} \pm \frac{\sqrt{15}}{2}$       c)  $\frac{1}{2} \pm \frac{\sqrt{3}}{6}$   
d)  $\frac{1}{2} \pm \frac{\sqrt{15}}{4}$       e)  $\frac{1}{3} \pm \frac{\sqrt{3}}{6}$       f) None of these

6) Solve for  $x$ :  $\sqrt[4]{3^{x+5}} = 9^{2x-7}$ .

- a)  $x = \frac{25}{7}$       b)  $x = 5$       c)  $x = \frac{61}{15}$       d)  $x = \frac{61}{24}$       e)  $x = \frac{61}{12}$       f) None of these

7) Given the following equation:  $x^3 + 3x^2 + 2x + 6 = 0$ . What is the sum of the real solution(s)?

- a)  $1 + \sqrt{2}$       b)  $-3$       c)  $-3 - \sqrt{2}$       d)  $-3 + \sqrt{2}$       e)  $1 - \sqrt{2}$       f) None of these

8) Find all solutions:  $\sqrt{3x+1} - 1 = x$ .

- a)  $x = 0$       b)  $x = \{0, 5\}$       c)  $x = \{0, 1\}$       d)  $x = \pm 1$       e)  $x = \{0, 2\}$       f) None of these

9) Solve the following inequality, with the solution written in interval notation:  $\frac{-3(-x-1)}{8} < \frac{7}{3}$ .

- a)  $(-\infty, \frac{53}{9})$       b)  $(\frac{53}{9}, \infty)$       c)  $(\frac{37}{9}, \infty)$       d)  $(\frac{47}{9}, \infty)$       e)  $(-\infty, \frac{47}{9})$       f) None of these

10) Solve  $\frac{3x-5}{x+3} \leq 2$  with the solution written in interval notation.

- a)  $[-11, 3)$       b)  $(-3, 11]$       c)  $(-\infty, -3) \cup (1, \infty)$       d)  $(-\infty, -3) \cup [11, \infty)$       e)  $[-1, 3)$

f) None of these

11) Find all  $x$  values that do NOT solve the inequality  $-23 \leq 3(1-4x) + 1 \leq 25$  and write the answer in interval notation.

- a)  $(-\infty, \frac{-7}{4})$       b)  $(\frac{-21}{4}, \frac{-9}{4}) \cup (\frac{9}{4}, \infty)$       c)  $(-\infty, \frac{-21}{4})$   
d)  $(-\infty, \frac{-7}{4}) \cup (\frac{9}{4}, \infty)$       e)  $(\frac{-7}{4}, \frac{9}{4})$       f) None of these

12) Given the function  $h(x) = x^2 - 4$ . Evaluate and simplify  $h(x) + 3 + h(x + 3)$ .

- a)  $2x^2 - 2$     b)  $x^2 + 2x + 4$     c)  $x^2 + 6x + 5$     d)  $2x^2 + 6x + 4$     e)  $x^2 + 4$     f) None of these

13) Which of the following represents a function of  $x$ ?

- a)  $y = 2$     b)  $x^2 + y^2 = 8$     c)  $x = |y| + 4$     d)  $y^2 + x = 1$     e)  $x = 1$     f) None of these

14) Find the quadratic equation whose  $x$ -intercepts are  $-7$  and  $6$ , and  $y$ -intercept is  $4$ .

- a)  $y = \frac{-2}{21}(x + \frac{1}{2})^2 + \frac{13}{21}$     b)  $y = \frac{-2}{21}x^2 + 4$     c)  $y = \frac{-2}{21}(x + \frac{1}{2})^2 + \frac{169}{42}$   
d)  $y = \frac{-2}{21}(x - \frac{1}{2})^2 + \frac{13}{21}$     e)  $y = \frac{-2}{21}(x + 7)^2 + 4$     f) None of these

15) A company finds that the revenue for selling  $x$  units of one of its products can be modeled by the quadratic function  $R(x) = -0.5x^2 + 200x$ , where  $R$  is in units of dollars. How many units of this product should be sold so that the maximum revenue is achieved?

- a) 400 units    b) 100 units    c) 500 units    d) 300 units    e) 200 units    f) None of these

16) A projectile is shot from a cannon on top of a 48-foot tall building. The height of the projectile as a function of time can be modeled by the quadratic function  $h(t) = -16t^2 + 32t + 48$ , where  $h$  is in units of feet. What is the maximum height?

- a) 48 feet    b) 32 feet    c) 72 feet    d) 64 feet    e) 96 feet    f) None of these

17) Let  $m(x) = -x$ ,  $a(x) = 2x + 3$ ,  $t(x) = x^2 - 4x + 5$ ,  $h(x) = 2x^3$ . Evaluate  $m(a(t(h(-1))))$ .

- a) 13    b) -18    c) -37    d) 18    e) 4    f) None of these

18)  $y^2$  varies directly as  $x$ . If  $x = 8$  when  $y = 6$ , find  $x$  when  $y = 3$ .

- a)  $1/2$     b) 4    c) 12    d)  $3/2$     e) 2    f) None of these

19) Solve the following equation for  $y$ :  $x = \frac{4y - 3}{6 - y}$

- a)  $y = \frac{6x + 3}{4 - x}$     b)  $y = \frac{6 - x}{x + 4}$     c)  $y = \frac{6x + 3}{x + 4}$     d)  $y = \frac{6x - 3}{4 - x}$     e)  $y = \frac{x - 6}{x - 4}$   
f) None of these

20) Find the sum of all zeros of  $P(x) = (x - 2)(x + 3)(7x^2 + 35x + 28)$ .

- a) -4    b) 1    c) -6    d) 4    e) -1    f) None of these

21) Let  $Y$  represent the yield on a bond, and let  $P$  represent the price. Assume that  $Y = k/P$  for some constant  $k$ . If the yield on the bond is 4% when the price is 100 dollars, find the yield when the price is 80 dollars.

- a) 2%    b) 4%    c) 6%    d) 5%    e) 8%    f) None of these

22) Find the value of  $x$ , given the following system of equations:

$$\begin{aligned} 6x - 15y &= 7 \\ 4x + 8 &= 10y. \end{aligned}$$

- a) 13    b) 37    c)  $-5/12$     d) No solution.    e) There are infinitely many values of  $x$ .    f) None of these

23) Suppose  $a, b, c$  are real numbers, and  $(a + b - 5)^2 + (b + 2c + 3)^2 + (c + 3a - 10)^2 = 0$ .

Find the integer closest to  $a^3 + b^3 + c^3$ .

- a) 47    b) 49    c) 51    d) 63    e) 57    f) None of these

24) A family has several children. Each boy in this family has as many sisters as brothers, but each of the girls has twice as many brothers as sisters. How many children are in this family?

- a) 6      b) 8      c) 4      d) 5      e) 7      f) None of these

25) Solve for  $x$ :  $x + \frac{x}{1+2} + \frac{x}{1+2+3} + \dots + \frac{x}{1+2+\dots+4041} = 4041$ .

**Hint:** For each  $n > 0$ ,  $\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$ .

- a) 2020      b) 2024      c) 2021      d) 2022      e) 2023      f) None of these

26) Suppose the following are true:  $x^2 = 17x + y$ ,  $y^2 = x + 17y$ ,  $x \neq y$ .

What is  $\sqrt{x^2 + y^2 + 1}$ ?

- a) 18      b) 17      c) 19      d)  $\sqrt{37}$       e)  $\sqrt{649}$       f) None of these

27) Suppose that  $x, y$  are real numbers with the following relationship:  $(x + \sqrt{1+x^2})(y + \sqrt{1+y^2}) = 1$ .

What is  $(x+y)^2$ ?

- a)  $\frac{1}{2}$       b) 1      c)  $-\frac{1}{2}$       d) 0      e) -1      f) None of these

28) The product of three natural numbers  $x, y, z$  is 192. Suppose  $z = 4$  and let  $p$  represent the average of  $x$  and  $y$ .

What is the minimum possible value of  $p$ ?

- a) 6      b) 7      c) 9.5      d) 8.5      e) 8      f) None of these

29) Suppose  $3b^2 + \frac{1}{b^2} = 4$ . Give the smallest positive value of  $8b^3 + \frac{1}{b^3}$ .

- a)  $\frac{27\sqrt{3}}{9}$       b) 9      c)  $\frac{35\sqrt{3}}{9}$       d) 8      e)  $\frac{29\sqrt{3}}{9}$       f) None of these

30) Tony and Steve can complete a job in 2 hours. Tony and Bruce can complete the same job in 3 hours. Steve and Bruce can complete the same job in 4 hours. Assuming each person works at a constant rate, whether working alone or working with others, how long, in minutes, will the same job take if Tony, Steve, and Bruce all work together?

Round your answer to the nearest integer.

- a) 111 minutes      b) 55 minutes      c) 28 minutes      d) 101 minutes      e) 89 minutes      f) None of these

31) In a class of  $a$  students, the average (arithmetic mean) of the test scores is 70. In another class of  $b$  students, the average score for the same test is 92. When the scores of the two classes are combined, the average of the test scores is 86. What is the value of  $\frac{a}{b}$ ?

- a) 3      b)  $\frac{8}{3}$       c) 4      d)  $\frac{3}{8}$       e)  $\frac{5}{16}$       f) None of these

32) How many positive integers for  $n$  satisfy the condition that  $n^2 + 45$  is a perfect square?

- a) 3      b) 1      c) 5      d) 2      e) 4      f) None of these

33) Natasha writes a 2025 term arithmetic sequence of positive integers, and Clint writes a different 2025 term arithmetic sequence of integers. Natasha's first term is the negative of Clint's first term. If the sum of the terms in each sequence is equal, what is the smallest possible value of the first term in Natasha's sequence?

- a) 1012      b) 42      c) 376      d) 45      e) 506      f) None of these

34) The 54th and 4th terms of an arithmetic sequence are -125 and 0 respectively. Find the 42nd term.

- a) 17      b) -100      c) 45      d) -95      e) 110      f) None of these

35) Suppose  $x, y$  are positive integers, and  $x + xy + y = 54$ ?. What is  $x + y$ ?  
a) 16                      b) 6                      c) 18                      d) 15                      e) 14                      f) None of these

36) Timmy and Tommy represented North High in a math contest. Timmy outscored Tommy by 16 points; as a team, they scored 80 points. David and Eli represented South High in the same contest. Eli outscored David by 10 points; as a team, they scored 60 points. Arrange the four students from highest score to lowest score.

a) Timmy, David, Tommy, Eli                      b) Tommy, Timmy, Eli, David                      c) Eli, Timmy, Tommy, David  
d) Timmy, Eli, Tommy, David                      e) Timmy, Tommy, Eli, David                      f) None of these

37) Find the 10th term of the sequence: 3, -6, 12, -24,...

a) -3072                      b) -512                      c) -2452                      d) -19683                      e) -1536                      f) None of these

38) A theater creates a specific arrangement for certain plays. The first row has 80 seats, the second row has 88, the third row has 96, and so on. If there is room for 25 rows, how many chairs are needed to set up the theatre?

a) 2400                      b) 4400                      c) 5600                      d) 3525                      e) 3850                      f) None of these

39) How many 4-digit numbers can be formed using the even digits 0,2,4,6,8 if repetitions are not allowed and the number must be less than 6000 and divisible by 10?

a) 18                      b) 16                      c) 24                      d) 12                      e) 6                      f) None of these

40) Solve for  $x$ :  $\left(x - \frac{1}{x}\right)^{\frac{1}{2}} + \left(1 - \frac{1}{x}\right)^{\frac{1}{2}} = x$

a)  $\frac{5}{4}$                       b)  $\frac{1 + \sqrt{5}}{2}$                       c)  $\frac{\sqrt{3}}{2}$                       d)  $\frac{1 + \sqrt{3}}{2}$                       e)  $\frac{1 + \sqrt{3}}{2}$                       f) None of these