

Algebra 2 Exam - University of Houston 2023 Math Contest
January 28, 2023

1) Find the sum of all divisors of 2023.

- a) 2612 b) 2848 c) 2456 d) 2352 e) 2536 f) None of these.

2) Given $\log_{10}(18) = a$, $\log_{10}(10) = b$ and $\log_{10}(3) = c$. Express $\log_{10}(120)$ in terms of a , b and c .

- a) $2a + b - 2c$ b) $2a + b - 3c$ c) $3a + 2b - 4c$
d) $2a + 2b - 4c$ e) $3a + 4b - 6c$ f) None of these.

3) $f(x) = x^3 - 27$. Give $f^{-1}(37)$.

- a) 4 b) 3 c) 6 d) 5 e) 2 f) None of these.

4) Give the positive product of the sum and difference of solutions to $|5x - 3| = 7$.

- a) $76/25$ b) $16/5$ c) $82/25$ d) $84/25$ e) $78/25$ f) None of these.

5) Which of the following is equivalent to $(3\sqrt{4} + 2)(\sqrt{4} + 1)$?

- a) $5\sqrt{4} + 15$ b) $8\sqrt{4} + 6$ c) $6\sqrt{4} + 11$ d) $6\sqrt{4} + 7$ e) $8\sqrt{4} + 8$ f) None of these.

6) Let $f(x) = 2\ln(x) + 3$ and $g(x) = x^2 + 3$. Find $g(f^{-1}(5))$.

- a) $e^3 + 3$ b) $e + 3$ c) $e^5 + 3$ d) $e^2 + 3$ e) $e^4 + 3$ f) None of these.

7) Let $x > 0$ and suppose the area of a rectangle is $x^3 + 3x^2 + 7x + 12$. One of the sides has length $x + 3$. What is the length of the other side?

- a) $x^2 + 7 - \frac{9}{x+3}$ b) $x^2 + 5 - \frac{4}{x+3}$ c) $x^2 + 2x + 4$ d) $x^2 + 3x + 7$

e) It is not possible to determine the length from the given information.

f) None of these.

8) The equation $0 = 3x^2 - 12x + 19$ has complex roots which can be written in the form $a + bi$ and $a - bi$. Give the value of $a^2 - b^2$.

- a) $5/3$ b) $13/6$ c) $9/5$ d) $7/3$ e) $7/4$ f) None of these.

9) Let a solve $\log_2(\log_5(\sqrt[4]{a+3})) = -2$. Give the x intercept of the line $ax + 3y = 5$.

- a) $-5/4$ b) $5/3$ c) $5/4$ d) $5/2$ e) $-5/3$ f) None of these.

10) Find the value of the y -intercept for the function $f(x) = 5(x+3)^2 - (x+14) + \sqrt{10-x}$.

- a) $27 + \sqrt{10}$ b) $1 + \sqrt{10}$ c) $15 + \sqrt{10}$ d) $34 + \sqrt{10}$ e) $31 + \sqrt{10}$ f) None of these.

11) A mixture containing 11% acid is mixed with 2 quarts of a mixture which is 16% acid to obtain a solution that is 13% acid. How much of the 11% solution must be used?

- a) $37/13$ quarts b) 4 quarts c) 13 quarts d) 3 quarts e) $21/13$ quarts f) None of these.

12) Amanda and Joe represented North High School in a math contest. Joe outscored Amanda by 12 points and their team scored 80 points in total. Lianne and George represented South High School in the same math contest. Lianne scored 3 times as many points as George and their team scored 72 points in total. Let x be the sum of the scores of Amanda and George, and y be the sum of the scores of Joe and Lianne. Give $y - x$.

- a) 42 b) 48 c) 38 d) 46 e) 54 f) None of these.

13) There are values of x and y that allow the system below to be solved for a and b :

$$a - 3xb = 1$$

$$2a + yb = y.$$

The values of x and y that cause the system to have no solution form a line with a point missing in the xy plane. Give the sum of the coordinates of the missing point.

- a) $2/3$ b) 2 c) $5/3$ d) $4/3$ e) $3/2$ f) None of these.

14) 843 guests attended a party, and 3 waiters took care of all the guests. Waiter 3 took care of twice as many guests as Waiter 1, and Waiter 2 took care of three times as many guests as Waiter 3. No guest was served by all three waiters, and 3 guests were each taken care of by two waiters, with none of the three guests served by the same 2 waiters. How many guests were taken care of by Waiter 2?

- a) 564 b) 582 c) 492 d) 426 e) 474 f) None of these.

15) It takes Georgia 40 minutes to walk 2 miles uphill from her house to the grocery store, and it takes her half as much time to walk downhill on the way back. Her friend Sami drives 12 times as fast as Georgia's average speed during her round trip when she drives to a restaurant that is 9 miles away. How long does it take Sami to get to the restaurant?

- a) 11.25 minutes b) 11.5 minutes c) 11 minutes d) 11.75 minutes e) 12 minutes f) None of these.

16) A mining company has two mines. One day's operation at mine #1 produces ore that contains 30 metric tons of copper and 600 kilograms of silver, while one day's operation at mine #2 produces ore that contains 40 metric tons of copper and 380 kilograms of silver. Suppose the company operates mine #1 for x days and mine #2 for y days. Find $x + y$ to produce 330 metric tons of copper and 5340 kilograms of silver.

- a) 9 b) 13 c) 10 d) 12 e) 11 f) None of these.

17) A rectangle is inscribed inside a circle of radius R . The perimeter of this rectangle is 24 units, and its area is 32 square units. What is the value of R ?

- a) $4\sqrt{2}$ b) $4\sqrt{5}$ c) 20 d) $2\sqrt{5}$ e) 80 f) None of these.

18) Suppose $b > 4$, and the solution to the inequality $\frac{x^2 - 8x + 12}{x - b} \geq 0$ is a half line that is missing a point. Give the value of b .

- a) 6 b) 12 c) 10 d) 8
e) There is not enough information to determine b .
f) None of these.

19) When $3x^4 + 20x^2 + 12$ is factored completely with integer coefficients, the sum of the coefficients of the sum of its factors is

- a) 18 b) 14 c) 10 d) 16 e) 12 f) None of these.

20) A rectangle has the property that when the length and width are each increased by 6 inches, the area increases by 86 square inches, and when the length and width are increased by 3 and 4 inches, respectively, the area increases by M square inches. The rectangle makes sense for all values of $b > M > a$ for some number $b > a > 0$. Give the largest possible value of $b - a$.

- a) $31/3$ b) $25/3$ c) 9 d) $28/3$ e) 12 f) None of these.

21) Determine the largest possible interval $[c, d]$ so that whenever $c < a < d$, the parabolas $y = 27x^2 + 10x + 11$ and $y = -23x^2 + ax + 3$ do not intersect. Give the value of d/c .

- a) $-2/3$ b) $-3/2$ c) $-5/6$ d) $-5/4$ e) $-5/3$ f) None of these.

22) Let $Q = (-1/2, 1/2)$. For $0 \leq x \leq 5$, the square of the distance from Q to $(x, 3)$ has a graph that is a portion of a parabola of the form $y = ax^2 + bx + c$. Give the sum of the coordinates of the vertex of the parabola $y = cx^2 + ax + b$.

- a) 14/13 b) 12/13 c) 23/26 d) 29/26 e) 25/26 f) None of these.

23) Find the equation of the parabola that passes through the points $(-6, 6)$, $(-4, 0)$ and $(0, 12)$. Give the sum of the coefficients of the parabola.

- a) 22 b) 19 c) 20 d) 18 e) 21 f) None of these.

24) Two positive numbers x and y have the property that their difference is 9 and the difference of their square roots is 2. What is the sum of these two numbers?

- a) 49/4 b) 97/8 c) 99/8 d) 195/16 e) 47/4 f) None of these.

25) The points $P = (-5, 4)$, $Q = (4, -2)$ and $R = (7, 1)$ lie on the graph of a function of the form $f(x) = a|x - b| + c$ for some values a , b and c . Give $a/b + c$.

- a) -3.1 b) -2.8 c) -3 d) -2.9 e) -3.2 f) None of these.

26) The domain of the function $g(x) = \sqrt{5 - 4x - x^2}$ is an interval of the form $[a, b]$. Give b/a .

- a) -0.2 b) -0.1 c) -0.3 d) -0.15 e) -0.25 f) None of these.

27) Let $f(x) = 2^{x-1}$ and $g(x) = f^{-1}(x + 1)$. Give the slope of the line that passes through the point $(-2, 3)$ and $(7, g(7))$.

- a) 2/11 b) 1/9 c) 2/13 d) 1/7 e) 1/12 f) None of these.

28) Let $f(x) = x^3 - 3x^2 + 1$ and $g(x) = 2x^2 + x$. The remainder of $f(x)$ divided by $g(x)$ has the form $ax + b$ for some numbers a and b . Give b/a .

- a) 5/17 b) 4/7 c) 4/9 d) 3/8 e) 2/5 f) None of these.

29) The graphs of $f(x) = 3x - 7$ and $g(x) = 2/(x + 1)$ intersect at two values of x of the form $a + \sqrt{b}$ and $a - \sqrt{b}$. Give b/a .

- a) 33/7 b) 29/2 c) 31/6 d) 11/2 e) 13/2 f) None of these.

30) Determine the values of a and b for which the system

$$2x + ay = 3$$

$$-7x + by = 11$$

either does not have a solution or has more than one solution, and then give the smallest value of b/a .

- a) 11/3 b) -7/11 c) 9/2 d) -7/2 e) 3/11 f) None of these.

31) Determine the value of a for which the system of equations

$$-x + 4y + 3z = -3$$

$$-3x + 13y + 12z = -7$$

$$4x - 14y - az = 15$$

does not have a solution. Then give the x coordinate of the vertex of the parabola $y = ax^2 + \frac{x}{a} + 3$.

- a) -1/6 b) -1/12 c) -1/36 d) -1/72 e) -1/24 f) None of these.

32) Find the point on the line $2x - 3y = 7$ that is closest to the point $(-3, 2)$, and give the quotient of the x and y coordinates.

- a) $1/65$ b) $2/39$ c) $3/97$ d) $1/31$ e) $2/39$ f) None of these.

33) Give the largest value of $x + 2y$, given that $x + 6y \leq 6$, $x \leq 5$ and $x, y \geq 0$.

- a) $16/3$ b) $19/3$ c) 6 d) $17/3$ e) 5 f) None of these.

34) Let $f(x) = 3x - 5$ and $g(x) = x^2 - 3x$. Give the x coordinate of the vertex of the parabola given by the graph of $h(x) = f(g(x)) - g(f(x))$.

- a) $5/4$ b) $3/2$ c) $7/2$ d) $5/2$ e) $7/4$ f) None of these.

35) One of the factors of $3x^3 - 2x^2 - 5x + 4$ is $x - 1$. Let $h(x)$ be the reciprocal of the product of the other two factors. Give the sum of the x coordinates of the vertical asymptotes of the graph of $h(x)$.

- a) $-5/3$ b) $1/3$ c) $-2/3$ d) $4/3$ e) $-1/3$ f) None of these.

36) The complex number $\frac{(2 - 3i)(3 + 5i)}{1 + 2i}$ can be written in the form $a + bi$. Give a/b .

- a) $-22/41$ b) $-19/21$ c) $-13/18$ d) $-23/41$ e) $-25/39$ f) None of these.

37) Integer points in the xy plane are points of the form (m, n) where both m and n are integers. Circles of radius $2^{-|n|}$ are centered at each integer point (m, n) along the lines $y = x$ and $y = -x$ for $|x| \leq 5$, except at the origin. Give the number of these circles.

- a) 11 b) 20 c) 10 d) 40 e) 21 f) None of these.

38) Give the sum of the circumferences of the circles in the previous problem.

- a) $29\pi/4$ b) $31\pi/4$ c) $15\pi/2$ d) $17\pi/2$ e) $33\pi/4$ f) None of these.

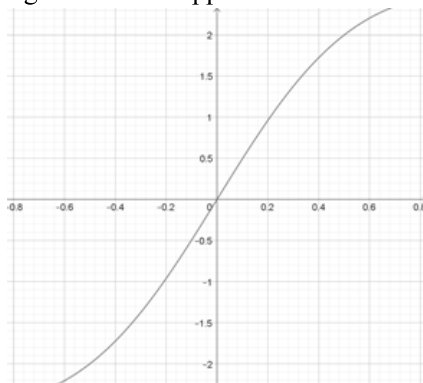
39) Give the sum of the squares of the x coordinates of the points on the graph of $|y| = x^2$ that lie on the circle of radius 5 centered at the origin.

- a) $2\sqrt{101} - 1$ b) $2\sqrt{101}$ c) $2\sqrt{101} - 2$ d) $2\sqrt{101} + 2$ e) $2\sqrt{101} + 1$
f) None of these.

40) Jenny is twice as old as her sister Mary's youngest daughter. This daughter is seven years younger than Mary's oldest son. Her son is the same age as Jenny's son Mark, who is 18 years younger than Jenny. Give the sum of the ages of Jenny, Mary's youngest daughter, Mary's oldest son, and Jenny's son Mark.

- a) 142 b) 131 c) 137 d) 139 e) 135 f) None of these.

41) f and g are functions, and numbers x and y satisfy $\sqrt{3f(x) + g(y) - 5} - \sqrt{3f(x) + g(y) - 25} = 6$ and $f(x) + g(y) = 30$. The graph of $f(x)$ is given below. Approximate x .



- a) -0.25 b) -0.3 c) -0.4 d) -0.36 e) -0.2 f) None of these.

42) a, b, c, d and e are real numbers, and the regression line for the data $(1, a), (2, b), (3, c), (4, d)$ and $(5, e)$ is given by $y = -x/2 + 2$. Give the value of $a + b + c + d + e$.

- a) 2.2 b) 2.4 c) 2.5 d) 1.8 e) 2.7 f) None of these.

43) The graph of the function $f(x) = 2ax^2 + 3bx + 4c$ has its vertex at $(-3, 4)$ and passes through the point $(1, 36)$. Find $a + b + c$.

- a) 11 b) 11.5 c) 11.5 d) 10 e) 10.5 f) None of these.

44) Give the average of the solutions to the equation $\sqrt{a^2 + 2a + 7} = 3a + 4$.

- a) -0.25 b) -0.5 c) -0.625 d) -0.75 e) -1.375 f) None of these.

45) Let S be the sum of the numbers excluded from the domain of the function $f(x) = \frac{x - 4}{x^2 - 5x + 4}$. Give the

value of $\frac{5S + 7}{S^2 + 2S - 5}$.

- a) -6 b) 4/5 c) 16/15 d) -4 e) 5 f) None of these.

46) 10 rectangles are located in the first quadrant with their bases on the x -axis. Each base has length $1/2$, and each height is $1/4$. The rectangles are equally spaced with the lower left-hand corner of the first rectangle at $(1/2, 0)$, the lower left-hand corner of the second rectangle at $(3/2, 0)$, and so forth. Define the function $g(x)$ for $0 \leq x \leq 10.5$ to be the sum of the areas of the rectangles and portions of rectangles with bases between x and 10.5 . Find $g(4.75)$.

- a) 5/8 b) 9/16 c) 3/4 d) 11/16 e) 1/2 f) None of these.

47) Let $g(x)$ be the function defined in the previous problem. Give the smallest number $a > 0$ so that $a(10.5 - x) \geq g(x)$ for all $0 \leq x \leq 10.5$.

- a) 3/16 b) 3/8 c) 5/16 d) 1/4 e) 1/8 f) None of these.

48) Let $g(x)$ be the function defined two problems above, and let a be the number found in the previous problem. Give the number of points of intersection of the graphs of $y = g(x)$ and $y = a(10.5 - x)$ for $0 \leq x \leq 10.5$.

- a) 15 b) Infinitely many c) 10 d) 12 e) 14 f) None of these.

49) Find the x -coordinates of the point or points on the graph of $y = 4x^2$ that are closest to the point $(0, 4)$. Give the sum of the quantity given by the square of the minimum distance plus the sum of the squares of the x -coordinates of the points.

- a) 169/64 b) 165/64 c) 187/64 d) 125/64 e) 145/64 f) None of these.

50) The area of the triangle with vertices $(12, 3), (7, 6)$ and $(-1, a)$ is 51. Give the sum of the absolute values of the possible values of a .

- a) 40.8 b) 42.8 c) 41.6 d) 42.4 e) 40.2 f) None of these.