Physics Exam - University of Houston 2023 Math Contest
January 28, 2023

Instructions: Throughout this test, unless otherwise specified, please use g as the acceleration due to gravity at the surface of the earth. Vectors $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ are unit vectors along x, y, and z, respectively, in a normal Cartesian coordinate system. Let G be the universal gravitational constant. To simplify calculations, you may use $g = 10$ m/s^2, and the value of the trigonometric functions at the following angles:

θ	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined

1) A drag racer starts from rest and accelerates uniformly in a straight line at a constant rate of 3.00 m/s^2 WEST for 6.00 s. He next maintains a constant velocity for 10.0 s. Then he accelerates at a steady rate of 2.00 m/s^2 EAST for 9.0 s. What is the racer's final speed?
a) 36.0 m/s b) 9.00 m/s c) 0.00 m/s d) 12.0 m/s e) 18.0 m/s

2) A drag racer starts from rest and accelerates uniformly in a straight line at a constant rate of 3.00 m/s^2 WEST for 6.00 s. He next maintains a constant velocity for 10.0 s. Then he accelerates at a steady rate of 2.00 m/s^2 EAST for 9.00 s. What is the racer's total displacement?
a) 261 m b) 153 m c) 207 m d) 315 m e) 234 m

3) A projectile is launched at an angle of $60.0°$ above the horizontal across the level ground with an initial speed of V_0 greater than zero. When the projectile is at the highest point of its trajectory:
a) The magnitude of its acceleration is zero b) Its speed is zero c) Its speed is non zero
d) The magnitude of its displacement is zero e) The magnitude of its average velocity is zero

4) A football is kicked at an angle of $60.0°$ above the horizontal across the level ground with an initial speed of $V_0 = 12.0$ m/s and lands at the same level from which it was struck. If the ball experiences no appreciable air resistance, how long will it take for the football to hit the ground?
a) $1.20\sqrt{3}$ s b) $2.40\sqrt{3}$ s c) $\frac{2.40}{\sqrt{3}}$ s d) 1.20 s e) 2.40 s

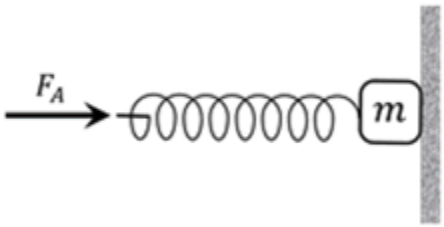
5) A football is kicked at an angle of $60.0°$ above the horizontal across the level ground with an initial speed of $V_0 = 12.0$ m/s and lands at the same level from which it was struck. If the ball experiences no appreciable air resistance, calculate the magnitude of the football's average velocity when it is in the air.
a) 12.0 m/s b) 6.0 m/s c) $12.0\sqrt{3}$ m/s d) 0.0 m/s e) $6.0\sqrt{3}$ m/s

6) A helicopter needs to travel North. It has an air speed of 90 km/hr and flies in a steady wind blowing to the west at 45 km/hr. At what angle should the helicopter head to fly North relative to the ground?
a) 30 degrees North of East b) 60 degrees North of East c) 60 degrees West of North
d) 30 degrees North of West e) 30 degrees West of North

7) A crate on a horizontal surface is attached to a rope that angles upward, as shown in the figure. The tension in the rope gradually increases. If the crate doesn't move, which of the following statements is not true while the tension is gradually applied?

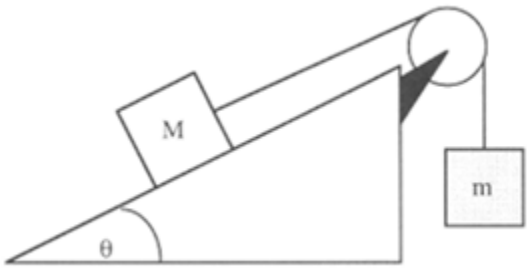
a) The magnitude of the maximum possible static friction force gradually decreases.
b) The magnitude of the static friction force gradually increases
c) The magnitude of the maximum possible static friction force remains constant.
d) There is no kinetic friction acting on the crate.
e) The magnitude of the normal force decreases

8) A spring with a spring constant of 200 N/m and block of mass 0.200 kg are held against a vertical wall as shown in the figure. What minimum distance must an applied force compress the spring from its relaxed length to keep the block from sliding down? The coefficient of static friction between the block and the wall is $\mu = 0.500$.



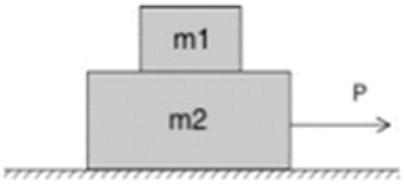
a) 2.0 cm b) 5.0 cm c) 3.0 cm d) 1.0 cm e) 4.0 cm

9) As shown in the figure, two blocks are connected by a massless rope over a frictionless pulley. The slope of the incline is $\theta = 30^\circ$. The coefficient of kinetic friction between block M and the table is $1/\sqrt{3}$. Block M has a mass of 2.00 kg, and block m has a mass of 6.00 kg. What would be the magnitude of the acceleration of the blocks if they were released from rest?



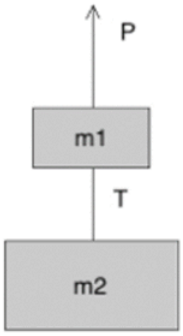
a) 4.00 m/s^2 b) 1.00 m/s^2 c) 5.00 m/s^2 d) 2.00 m/s^2 e) 3.00 m/s^2

10) A 4.00 kg (m_1) box sits atop a 6.00 kg box (m_2) on a perfectly smooth horizontal floor. When a horizontal pull to the right $P = 200$ N is exerted on the lower box, as shown in the figure, both boxes move together. What is the value of the friction between the two boxes?



a) 100 N b) 20.0 N c) 80.0 N d) 40.0 N e) 60.0 N

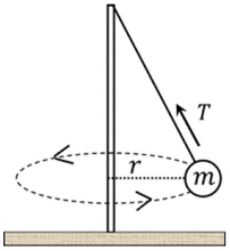
11) Two blocks are connected by a massless wire and pulled upward at constant speed by a pull P, as shown in the figure. Which one of the following relationships between the tension in the wire T, the pull P, Block 1 mass (m_1), and Block 2 mass (m_2) must be true?



a) $P - T = g(m_1)$ b) $P - T = g(m_2)$ c) $P + T = g(m_2)$
d) $P - T = g(m_1 + m_2)$ e) $T + P = g(m_1 + m_2)$

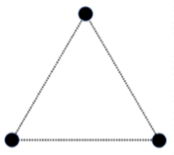
12) An object experiences uniform circular motion. Which of the following statements about it is true?
a) The velocity of the object is constant.
b) There is a constant magnitude force applied to the object.
c) The velocity of the object always points in the radial direction.
d) The acceleration of the object is in the same direction as its velocity.
e) The acceleration of the object is in the tangential direction.

13) A tether ball of mass $m = 2.0$ kg hangs from the top of a pole and swings around it in a horizontal circular path of radius $r = 10.0$ m, as shown in the figure. If the tangential speed of the ball is $v = 10.0$ m/s, what would the tension in the rope be?



a) $\sqrt{200}$ N b) $\sqrt{800}$ N c) $\sqrt{400}$ N d) $\sqrt{600}$ N e) $\sqrt{1000}$ N

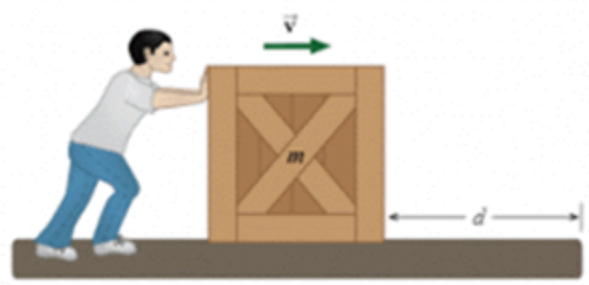
14) Three masses that are m kilograms each are held on the sides of an equilateral triangle of length L meters, as shown in the figure. If one of the masses is released while the other two are held in place, what is its initial acceleration if the only force acting on it are the gravitational forces due to the other two masses?



a) $2\sqrt{3}\dfrac{Gm}{L^2}$ b) $\sqrt{3}\dfrac{Gm}{L^2}$ c) $1\dfrac{Gm}{L^2}$ d) $\dfrac{\sqrt{3}}{2}\dfrac{Gm}{L^2}$ e) $2\dfrac{Gm}{L^2}$

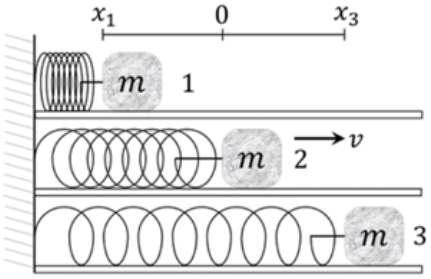
15) A force is applied to an ideal spring (initially in its equilibrium position) and does 200 J of work, stretching it 20 cm. How much work is required to stretch the spring by 40 cm from its equilibrium position?
a) 200 J b) 800 J c) 400 J d) 600 J e) 1600 J

16) A student pushes an 8.00 kg crate across the classroom floor a distance of 6.00 m before coming to rest. If the crate has an initial speed of 4.00 m/s, how much work did the student do on the crate? The static and kinetic friction coefficients between the crate and surface are 0.800 and 0.500.



a) 304 J b) 240 J c) 176 J d) 64.0 J e) 364 J

17) The figure shows a mass attached to a spring on top of a track at three different positions during its motion. The mass starts at position 1, moves to position 2, and ends at position 3. At position 1, the block is momentary at rest, and the spring is compressed to a distance X_1 from its relaxed length. At position 2, the spring had been released from its compressed state, and the mass traveled to the spring's relaxed length at $X = 0$. At position 3, the moving mass has come to a momentary stop after stretching the spring to a maximum distance of X_3 from its relaxed length. Which of the following statements is true if the friction between the mass and the track cannot be ignored in the system?

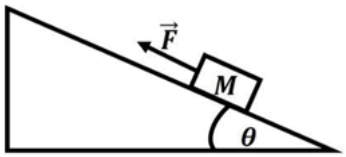


a) The distance from X_1 to 0 equals the distance from 0 to X_3.
b) The total mechanical energy is greater at position 2 than at position 3.
c) The total mechanical energy is the same at all positions.
d) The potential energy of the spring decreases between positions 2 and 3.
e) The acceleration of the mass is greatest at position 2.

18) How fast would a 60.0 kg person run to have kinetic energy equal to the amount of energy used by a 100 W light bulb in 1.00 minutes?

a) 10 m/s b) $10\sqrt{6}$ m/s c) $10\sqrt{2}$ m/s d) 20 m/s e) $10\sqrt{8}$ m/s

19) A UPS driver pulls a 20.0 kg box up the inclined frictionless surface for a distance of 2.00 m. The surface rises θ = 30.0 degrees above the horizontal, as shown in the figure. How much work does the driver do on the crate?

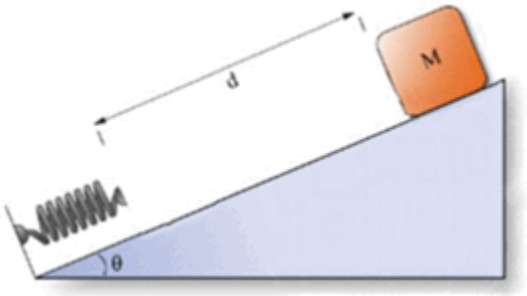


a) $200\sqrt{3}$ J b) 200 J c) 400 J d) $\frac{400}{\sqrt{3}}$ J e) $400\sqrt{3}$ J

20) A 30.0 kg block slides on a smooth horizontal surface at a constant speed of 10 m/s when it encounters an extended rough patch with a coefficient of kinetic friction of 0.500. After crossing the path, the block's velocity halved. Find the patch's length.

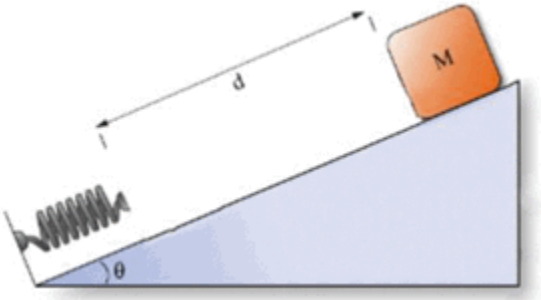
a) 60.0 m b) 15.0 m c) 7.50 m d) 30.0 m e) 45.0 m

21) A block of mass 5.00 kg is sitting on a frictionless ramp with a spring at the bottom, as shown in the figure. The ramp's angle with respect to the horizontal is θ = 30.0 degrees, and the spring has a spring constant of 200 N/m. The block, starting from rest, slides down the ramp a distance d before hitting the spring. Then, the spring compressed a distance of 50 cm as the block came to momentary rest. Find the distance d the block traveled before hitting the spring.



a) 2.00 m b) 1.50 m c) 2.50 m d) 0.50 m e) 1.00 m

22) A block of mass 5.00 kg is sitting on a frictionless ramp with a spring at the bottom, as shown in the figure. The ramp's angle with respect to the horizontal is θ = 30.0 degrees, and the spring has a spring constant of 200 N/m. The block, starting from rest, slides down the ramp a distance d before hitting the spring. Then, the spring compressed a distance of 50 cm as the block came to momentary rest. After the block comes to rest, the spring pushes the block back up the ramp. How fast is the block moving right after it comes off the spring?



a) $\sqrt{5.00}$ m/s b) $\sqrt{15.00}$ m/s c) $\sqrt{2.00}$ m/s d) $\sqrt{3.00}$ m/s e) $\sqrt{10.00}$ m/s

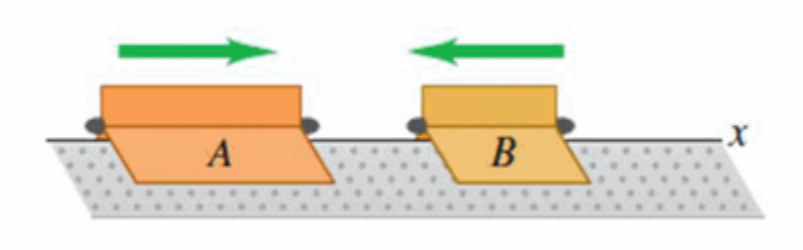
23) Calculate the final speed of a 100 kg rugby player who is initially running at 8.00 m/s but collides head-on with a padded goalpost and experiences a backward average force of 10000 N for 50.0 ms.

a) 6.00 m/s b) 3.00 m/s c) 7.00 m/s d) 4.00 m/s e) 5.00 m/s

24) Two astronauts, of masses 60.0 kg and 80.0 kg, are initially right next to each other and at rest in outer space. They suddenly push each other apart. After their interaction, the lighter astronaut moved a distance of 16.0 m in 12.0 s. How far has the heavier astronaut moved in the same amount of time?

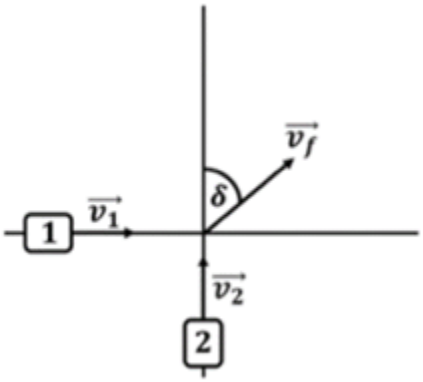
a) 12.0 m b) 14.0 m c) 10.0 m d) 8.0 m e) 6.0 m

25) Two gliders move toward each other at the same speed v on a frictionless air track. Glider B has a mass m, and glider A has twice the mass of glider B. What would be the kinetic energy of the gliders, in terms of the speed v and mass m, after they collide if the collision between the gliders is completely inelastic?



a) $\frac{1}{3}mv^2$ b) $\frac{1}{6}mv^2$ c) $\frac{1}{4}mv^2$ d) $\frac{1}{2}mv^2$ e) $\frac{1}{8}mv^2$

26) The figure shows two identical 5.00 kg blocks colliding in a completely inelastic collision at the origin. After the collision, the two objects stick together and move with a velocity of v_f = 40.0 m/s, as indicated in the figure. The value for angle δ is 30.0 degrees. How fast was mass 2 initially traveling?



a) $40.0\sqrt{3.00}$ m/s b) 80.0 m/s c) $20.0\sqrt{3.00}$ m/s d) 20.0 m/s e) 40.0 m/s

27) A 3.00 kg cart is attached to a horizontal spring that oscillates on a frictionless air track with a frequency of 5.00 Hz. What would be the length of a simple pendulum with the same period as the cart-spring system?

a) $\frac{1}{25\pi^2}$ m b) $\frac{1}{15\pi^2}$ m c) $\frac{1}{10\pi^2}$ m d) $\frac{1}{5\pi^2}$ m e) $\frac{1}{20\pi^2}$ m

28) A molecule of salt vibrates like a simple harmonic oscillator. Suppose the maximum speed of the molecule is 300 m/s, and its maximum acceleration is 3.00×10^{15} m/s^2; what is the frequency of its oscillations?

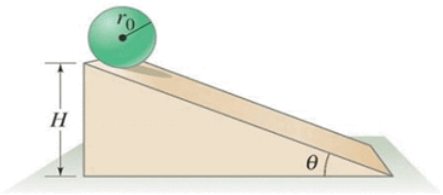
a) $\frac{1.00 \times 10^{13}}{2\pi}$ Hz b) $2.00 \times 10^{13}\,\pi$ Hz c) $\frac{9.00 \times 10^{17}}{2\pi}$ Hz

d) $\frac{9.00 \times 10^{15}}{2\pi}$ Hz e) $1.80 \times 10^{18}\,\pi$ Hz

29) A barrel rolls down a ramp. Which force produces a torque on the barrel?
a) The component of weight perpendicular to the ramp. b) The weight.
c) Friction. d) The normal force. e) The component of weight parallel to the ramp.

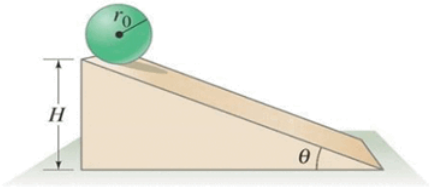
30) A bicycle has a front gear with a radius of 10.0 cm and a rear gear with a radius of 5.00 cm. How fast does the rear gear rotate if the front gear rotates at 24.0 rad/s?
a) 24.0 rad/s b) 48.0 rad/s c) 36.0 rad/s d) 12.0 rad/s e) 50.0 rad/s

31) A uniform solid sphere with a mass of 5.00 kg and a radius of 0.500 m rolls down a ramp 7.00 m long angled at 30.0° from the horizontal (without slipping), starting from rest. The moment of inertia of the solid sphere is $I = \frac{2}{5}MR^2$. What is the speed of the sphere's center of mass at the bottom of the ramp?



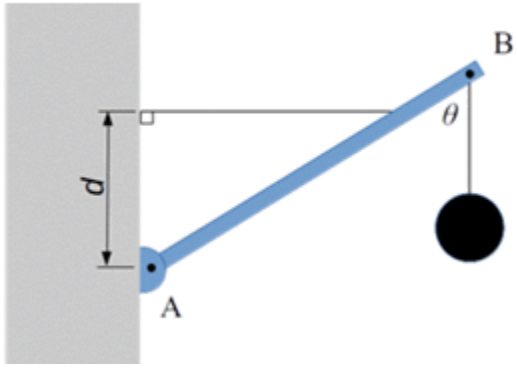
a) $\sqrt{350.0}$ m/s b) $\sqrt{70.0}$ m/s c) 7.00 m/s d) $\sqrt{50.0}$ m/s e) $\sqrt{\frac{50.0}{7}}$ m/s

32) A uniform solid sphere with a mass of 5.00 kg and a radius of 0.500 m rolls down a ramp 10.0 m long angled at 30.0° from the horizontal (without slipping), starting from rest. The moment of inertia of the solid sphere is $I = \frac{2}{5}MR^2$. What is the acceleration of the center of mass of the sphere?



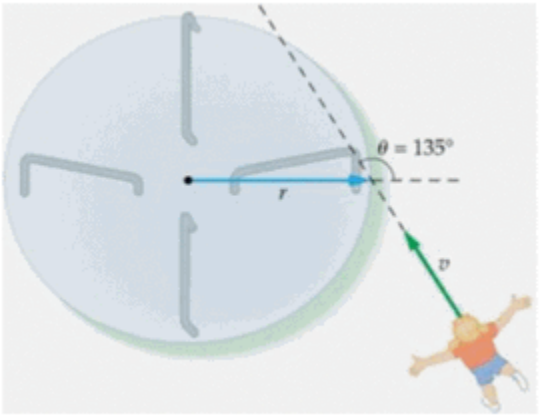
a) $\frac{50}{7}$ m/s^2 b) $\frac{5}{7}$ m/s^2 c) 14 m/s^2 d) $\frac{25}{7}$ m/s^2 e) $\frac{10}{7}$ m/s^2

33) A uniform metal rod (mass 2.00 kg) is attached to a hinge affixed to a wall at point A. A horizontal cable is connected to the wall $d = \sqrt{3.00}$ m above the hinge (point A) on one end and attached to the rod at the other. A ball of mass 3.00 kg is hung from a massless cable attached to the metal rod at point B. The angle between the rod and the cable holding the ball is $\theta = 60.0^\circ$. The distance between point A and point B is 5.00 m. What is the tension in the horizontal cable? (The rod is free to rotate about point A)



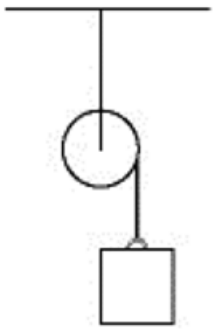
a) $\frac{100}{\sqrt{3.00}}$ N b) $\frac{50}{\sqrt{3.00}}$ N c) $50\sqrt{3.00}$ N d) 100 N e) 50 N

34) A 25.0 kg child runs with a speed of 4.00 m/s to the rim of a stationary merry-go-round. The direction of motion of the child makes an angle θ = 135 degrees with respect to the vector **r**, as shown in the figure. The merry-go-round has a moment of inertia of 500 kg m^2 and a radius of 2.00 m. The entire system begins to rotate when the child jumps onto the merry-go-round. What is the angular speed of the system (merry-go-round + child)? Ignore friction and any other type of external torque.



a) $\frac{\sqrt{2.00}}{12}$ rad/s b) $\frac{\sqrt{2.00}}{6}$ rad/s c) $\frac{1}{6\sqrt{2.00}}$ rad/s d) $\frac{\sqrt{5.00}}{12}$ rad/s e) $\frac{1}{12\sqrt{2.00}}$ rad/s

35) In the figure, a mass of 20.0 kg is attached to a light string wrapped around a cylindrical spool of 40.0 kg mass and 10.0 radius (the inertia of the cylindrical spool is equal to the inertia of a solid cylinder $I = \frac{1}{2}mr^2$). The spool is suspended from the ceiling, and the mass is then released from rest at a distance of 10.0 m above the floor. How long does it take to reach the floor?



a) 1.00 s b) 2.00 s c) 3.00 s d) 4.00 s e) 5.00 s

36) A new solar system has been discovered, where two planets orbit a star. The star has a mass of 12*MSun. Planet 1 has a mass of 12*ME, a radius of 3*RE, and an orbital radius of 0.500 AU. Planet 2 has a mass of 9*ME, a radius of 4*RE, and an orbital radius of 1.00 AU, where MSun is the mass of the Sun, ME is the mass of the Earth, RE is the radius of the Earth, and 1.00 AU is the orbital radius of Earth around the Sun. Which planet has the greatest acceleration due to gravity at its surface, which has the greater period about the star, and which has the greater angular momentum about the star?

a) Planet 2, Planet 1, Planet 2. b) Planet 1, Planet 2, Planet 2. c) Planet 2, Planet 2, Planet 1.
d) Planet 1, Planet 2, Planet 1. e) Planet 1, Planet 1, Planet 2.