

Precalculus Exam - University of Houston 2023 Math Contest
January 28, 2023

1) Given: $f(2x + 5) = 2x^3 - x^2 + 6$ find $f(1)$
a) 12 b) -30 c) -14 d) 5 e) none of the other answer choices provided

2) If the graph of $f(x) = (m - 10)x^3 + x^{n-6} + 2x + 1$ is a parabola then what is the value of $m - n$?
a) 18 b) -2 c) 12 d) 2 e) none of the other answer choices provided

3) Line L contains the points (2,1) and (1,-2), and Line M is perpendicular to Line N and goes through the point (2,4). What is the x -coordinate of the point of intersection of lines L and M?

a) $-\frac{29}{10}$ b) $\frac{29}{10}$ c) $\frac{9}{8}$ d) $\frac{19}{18}$ e) none of the other answer choices provided

4) If $y = 6x - 1$ and $y = 5 - \frac{x}{a}$ are the equations of parallel lines, then what is the value of a ?

a) 6 b) -6 c) $\frac{1}{6}$ d) $-\frac{1}{6}$ e) none of the other answer choices provided

5) Find the value of the expression

$$\sin \left[\sin^{-1} \left(\frac{-4}{5} \right) - \tan^{-1} \left(\frac{3}{4} \right) \right]$$

a) 1 b) $\frac{4}{3}$ c) -1 d) 0 e) $-\frac{3}{4}$ f) none of the other answer choices provided

6) Find the value of the expression

$$\cos^2 \left(\frac{7\pi}{12} \right) + \sin^2 \left(\frac{\pi}{12} \right)$$

a) $\frac{2 + \sqrt{3}}{2}$ b) $\frac{2 - \sqrt{3}}{2}$ c) $\frac{\sqrt{3} - 2}{2}$
d) $\frac{2\sqrt{3}}{2}$ e) $\frac{2 - \sqrt{3}}{3}$ f) none of the other answer choices provided

7) Find the value of $\sqrt{3} \cos(15^\circ) - \sin(15^\circ)$

a) $\sqrt{2}$ b) 1 c) $3\sqrt{2}$ d) $\sqrt{3}$ e) none of the other answer choices provided

8) A motorized sailboat leaves Galveston Bay, bound for Cozumel, 150 miles away. Maintaining a constant speed of 15 miles per hour, but encountering heavy crosswinds and strong currents, the crew finds, after 4 hours, that the sailboat is off course by 20° . How far is the sailboat from Cozumel at this time?

a) 95.8 miles b) 9185.53 miles c) 45 miles d) 105.13 miles
e) none of the other answer choices provided

9) Find the y -coordinate of the vertex of the parabola $y = -x^2 + 4x + 1$.

a) 5 b) -5 c) 2 d) 0 e) none of the other answer choices provided

10) Find the exact value of $\sec(\tan^{-1}(\sqrt{3})) + \csc\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right]$.
 a) 4 b) 2 c) 6 d) π e) none of the other answer choices provided

11) If $\sin(x) = \frac{-1}{3}$ and $\pi < x < \frac{3\pi}{2}$ find the exact value of $\cos(2x)$.
 a) $\frac{-7}{9}$ b) $\frac{6}{9}$ c) $\frac{9}{7}$ d) $\frac{7}{9}$ e) none of the other answer choices provided

12) Simplify the following expression: $\frac{(\tan(x) + 1)(\tan(x) + 1) - \sec^2(x)}{\tan(x)}$.
 a) 2 b) $2\sin(x)$ c) 1 d) $\tan(x)$ e) none of the other answer choices provided

13) Given: $17x = \pi$, evaluate the value of $\frac{\cos(8x)\cos(6x)}{\cos(15x) + \cos(3x)}$
 a) $\frac{1}{2}$ b) $\frac{-1}{2}$ c) 2 d) -1 e) 0

14) Which of the following is the period of the function $f(x) = 5\cos\left(\frac{x}{2} + 3\right) + 1$?
 a) 6π b) 2π c) 4π d) 3π e) π f) none of the other answer choices provided

15) Consider the equation $\frac{\cot(x) + \frac{\cos^3(x) + \sin^2(x)\cos^3(x)}{1 - \sin^4(x)} - \cos(x)}{\sin(x)} = 0$. Find the number of the solutions of the equation.
 a) 1 b) 3 c) 4 d) 0 e) 2

16) Consider $\triangle ABC$ with $\angle CAB = 121^\circ$, $|AC| = 5$ in, $|AB| = 8$ in. Find the area of the triangle.
 a) 47 in^2 b) 18.1 in^2 c) 19.2 in^2 d) 17.1 in^2 e) none of the other answer choices provided

17) Find the smallest solution for the equation $\cos(x) + \sin(x) = 1$ on $[0, 2\pi)$
 a) $\frac{\pi}{2}$ b) 0 c) $\frac{\pi}{6}$ d) $\frac{3\pi}{2}$ e) none of the other answer choices provided

18) Find the domain of the function $f(x) = \sqrt{1 - \log(15 - 5x)}$.
 a) (1, 3) b) [1, 3) c) (0, 3] d) [1, ∞) e) none of the other answer choices provided

19) Find the radius of the circle $x^2 + y^2 + 12x - 14y + 84 = 0$.
 a) 1 b) 6 c) 3 d) 8 e) none of the other answer choices provided

20) At one farm, trees are harvested once they reach a height of 20 ft. Suppose a farm worker determines that distance along the ground from his position to the base of a tree is 22 ft. He then uses an instrument called a clinometer held at an eye level of 6 ft to measure the angle of elevation to the top of the tree at 30.2° . What is the height of the tree? Is it tall enough to harvest?
 a) 25 ft, Yes b) 10 ft, Yes c) 18.8 ft, No d) 19 ft, No e) none of the other answer choices provided

21) If $\vec{a} = \langle 1, 2 \rangle$ and $\vec{b} = \langle 3, 3 \rangle$, what is the magnitude of the vector $2\vec{a} - \vec{b}$?
 a) $2\sqrt{3}$ b) $\sqrt{2}$ c) $2\sqrt{2}$ d) $3\sqrt{2}$ e) none of the other answer choices provided

22) Find the number of points where the circle $x^2 + y^2 = 100$ and the parabola $y^2 + 6x = 10$ intersects.
a) 0 b) 4 c) 1 d) 3 e) none of the other answer choices provided

23) If $ax^3 + bx^2 + cx + d = (x^2 - 1)(mx + n) + 3x + 4$ what is $b + d$?
a) 1 b) 5 c) 4 d) 2 e) 3

24) Solve $2^{2x} + 2^{x+2} - 12 = 0$.
a) 1 b) 3 c) 7 d) 2 e) none of the other answer choices provided

25) How many solutions does $4\cos^2(\theta) - 3 = 0$ have on $0 \leq \theta < 2\pi$?
a) 0 b) 2 c) 4 d) 3 e) none of the other answer choices provided

26) Identify the graph of the equation $4x^2 + y^2 - 8x + 4y + 4 = 0$.
a) ellipse b) parabola c) hyperbola d) circle e) none of the other answer choices provided

27) If $\frac{\tan(4x) - \tan(3x)}{1 + \tan(4x)\tan(3x)} = \frac{1}{2}$, what is the value of $\tan(2x)$?
a) -1 b) $\frac{3}{4}$ c) $-\frac{3}{4}$ d) $\frac{4}{3}$ e) none of the other answer choices provided

28) Which of the following is **not** a solution of the equation $\tan(2\theta) + 2\cos(\theta) = 0$ on the interval $0 \leq \theta < 2\pi$?
a) $\frac{3\pi}{2}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{6}$ d) $\frac{7\pi}{6}$ e) none of the other answer choices provided

29) The vector $\vec{v} = 4\vec{i} - 4\vec{j}$ find the direction of angle of \vec{v} .
a) 60° b) 215° c) 315° d) 45° e) none of the other answer choices provided

30) Adorning the top of the Board of Trade building in Chicago is a statue of Ceres, the Roman goddess of wheat. From street level, tower observations are taken 400 feet from the center of the building. The angle of elevation to the base of the statue is found to be 55.1° and the angle of elevation to the top of the statue is 56.5° . What is the height of the statue?
a) 33 b) 32 c) 31 d) 30 e) none of the other answer choices provided

31) Simplify $\frac{\cos^2(a) - \cos^2(b)}{\sin(a+b)\sin(a-b)}$.
a) -1 b) $\tan(a)$ c) $\tan(2a)$ d) 1 e) none of the other answer choices provided

32) Let $f(x) = 32e^{2x-1}$, $g(x) = 2\log_3(20x + 31)$ and $h(x) = \frac{4x-1}{2x+1}$. Compute $(f^{-1} \circ g \circ h^{-1})(3/2)$.
a) $\frac{1 + \ln(4)}{2}$ b) $\ln(1/4) + 1/2$ c) $1 + \ln(4)$ d) $\frac{1 - \ln(4)}{2}$ e) $\ln(1/4) - 1$
f) none of the other answer choices provided

33) Let $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ and $a = (\cos(\theta))^{\cos(\theta)}$, $b = (\sin(\theta))^{\cos(\theta)}$, $c = (\cos(\theta))^{\sin(\theta)}$, $d = (\cos(\theta))^{\sin(4\theta)}$. Which of the following is/are true?
I. $a < b < c$
II. $c < a < b$
III. $c < b < d$
IV. $d - a < 0$
a) II and III only b) II only c) II and IV only
d) I and III only e) III only f) none of the other answer choices provided

34) Let x be a real number satisfying $\cos(x) = \tan(x)$. Find the value of $\left(\frac{2}{\sin(x)} + \cos^4(x) - \sin(x)\right)^2$.

- a) 4 b) 9 c) 18 d) 16 e) 3
 f) none of the other answer choices provided

35) Given $M = \sin\left(2 \arcsin\left(\frac{3}{5}\right)\right) + \sec\left(2 \arccos\left(\frac{\sqrt{2}}{3}\right)\right)$. Find the value of $\sqrt{1 - M^2}$.

- a) $-\frac{31}{25}$ b) $\frac{269}{225}$ c) $\frac{2\sqrt{46}}{25}$ d) $-\frac{11}{25}$ e) $\frac{19}{225}$
 f) none of the other answer choices provided

36) Given $\frac{\cot(x)}{2 \tan(x) + 2 \cot(x)} = \frac{4 \sin(x) - 3}{2}$. Find the value of $\sin^2(x)$.

- a) $12 - 2\sqrt{2}$ b) $12 - 4\sqrt{2}$ c) $12 - 8\sqrt{2}$
 d) $12 + 4\sqrt{2}$ e) $8 - 4\sqrt{2}$ f) none of the other answer choices provided

37) Let $A = \frac{1 + \cos(40^\circ)}{\cos(55^\circ) \cdot \cos(35^\circ)}$ and $B = \frac{\tan(75^\circ)}{\sin(25^\circ)} - \frac{1}{\cos(25^\circ)}$. Which of the following is equivalent to $\frac{A \cdot B}{8(\sin^2(10^\circ) + \sin^2(100^\circ))}$?

- a) $\cos(40^\circ) \cos(75^\circ)$ b) $2 \cos(40^\circ) \sec(75^\circ)$ c) $4 \cos(20^\circ) \sec(75^\circ)$
 d) $2 \cos(40^\circ) \cos(25^\circ)$ e) $\cos(20^\circ) \sec(75^\circ)$ f) none of the other answer choices provided

38) Define the function $f(x) = \frac{2x^5 + 2x^4 + 50x^2 + 40x + 20}{2x^5 - 16x^3 + 4x^2 - 16x + 8}$. Consider the x -coordinates of points where this function intersects its horizontal asymptote. Find the sum of the squares of the x -coordinates of all of these points.

- a) $14 - 8\sqrt{3}$ b) $8 - 2\sqrt{3}$ c) $14 + 8\sqrt{3}$
 d) 12 e) 14 f) none of the other answer choices provided

39) Which of the following is an x -intercept for the function $f(x) = \frac{20 - 80 \cos^2\left(\frac{x}{2} + \frac{\pi}{6}\right)}{7 + 7 \sin(x)}$?

- a) $x = \frac{14\pi}{3}$ b) $x = 4\pi$ c) $x = \frac{29\pi}{6}$
 d) $x = \frac{13\pi}{3}$ e) $x = \frac{13\pi}{4}$ f) none of the other answer choices provided

40) Given $\cos(4x) = \frac{1}{5}$. Find the value of M that satisfies the equation $\sin^8(x) - \cos^8(x) - M \cos(2x) = 0$.

- a) -3.2 b) -2.4 c) -0.8 d) -0.4 e) -1.6 f) none of the other answer choices provided

41) Given x is an acute angle with $\cot(x) = 3$, evaluate $\frac{\cos(4x)}{\sin(3x)}$.

- a) $\frac{7\sqrt{10}}{65}$ b) $\frac{7\sqrt{10}}{13}$ c) $\frac{5\sqrt{10}}{7}$
 d) $\frac{2\sqrt{10}}{13}$ e) $\frac{3\sqrt{10}}{65}$ f) none of the other answer choices provided

42) Consider the solutions over the interval $[-\pi, 0]$ for the equation $\cos\left(2x - \frac{\pi}{3}\right) - \sin\left(x - \frac{\pi}{4}\right) = 0$. Find the sum of all solutions in this interval.

- a) $-\frac{11\pi}{36}$ b) $-\frac{\pi}{12}$ c) $-\frac{61\pi}{36}$ d) $-\frac{5\pi}{36}$ e) $-\frac{5\pi}{12}$ f) none of the other answer choices provided

43) Assume that $\tan(2^\circ) = y$. Which of the following is equivalent to the expression below?

$$\frac{\tan(1^\circ)\tan(61^\circ)}{\tan(3^\circ)\tan(31^\circ)} + \frac{\tan(61^\circ) + \tan(31^\circ)}{1 - \tan(61^\circ)\tan(31^\circ)}$$

a) $\frac{y+1}{y}$

b) $\frac{y-1}{y}$

c) $\frac{1-y}{y}$

d) $\frac{1-y}{y^2}$

e) $\frac{1-y}{y+1}$

f) none of the other answer choices provided