

Calculation Exam

University of Houston Math Contest 2024

Notes:

- Some problems are easier than others, and the problems are NOT listed in order of difficulty. There is no penalty for guessing.
- A few questions can be answered without computing tools.
- Some questions are related to previous questions.
- **Unless otherwise requested, select an answer that is closest to the exact answer, unless there is no answer within 10^{-4} of the exact answer.**
- If no such value appears, select "None of these."

1. Find the largest solution to the equation

$$\frac{1}{16}x^4 - 14x^3 - 15x^2 - 10x - 7 = 0.$$

- a. 225.0693
- b. 225.0611
- c. 225.0697
- d. 225.0699
- e. 225.0695
- f. None of these.

2. Find the smallest solution to the equation

$$\frac{1}{16}x^4 - 14x^3 - 15x^2 - 10x - 7 = 0.$$

- a. -0.8941
- b. -0.8943
- c. -0.8945
- d. -0.8947
- e. -0.8949
- f. None of these.

3. Give the slope of the line containing the points (101,23) and (317,42).

- a. 0.0879
- b. 0.0877
- c. 0.0875
- d. 0.0873
- e. 0.0881
- f. None of these.

4. Give the slope of the line perpendicular to the line containing the points (101,23) and (317,42).
- a. -11.3684
 - b. -11.3686
 - c. -11.3688
 - d. -11.3682
 - e. -11.3690
 - f. None of these.
5. Give the distance from the point (17,6) to the line $3x - 14y = 12$.
- a. 3.1427
 - b. 3.1425
 - c. 3.1429
 - d. 3.1423
 - e. 3.1421
 - f. None of these.
6. Give the sum of the coordinates of the point on the line $3x - 14y = 12$ that is closest to the point (17,6).
- a. 20.5855
 - b. 20.5857
 - c. 20.5851
 - d. 20.5853
 - e. 20.5859
 - f. None of these.

7. Solve the system

$$\begin{cases} 12x - 13.1y = 25 \\ -32x + 5y = 61 \end{cases}$$

and give the value of $x + y$.

- a. -6.8379
 - b. -6.8377
 - c. -6.8381
 - d. -6.8383
 - e. -6.8385
 - f. None of these.
8. Give the x -coordinate of the vertex of the parabola

$$y = -37x^2 + 161x - 23.$$

- a. 2.1759
- b. 2.1761
- c. 2.1757
- d. 2.1755
- e. 2.1753
- f. None of these.

9. Give the y -coordinate of the vertex of the parabola

$$y = -37x^2 + 161x - 23.$$

- a. 152.1419
- b. 152.1421
- c. 152.1423
- d. 152.1425
- e. 152.1427
- f. None of these.

10. Give the sum of the first 317 odd natural numbers.

- a. 100486
- b. 100487
- c. 100488
- d. 100489
- e. 100490
- f. None of these.

11. Give the sum of the first 317 even natural numbers.

- a. 100806
- b. 100807
- c. 100808
- d. 100809
- e. 100805
- f. None of these.

12. Let

$$f(x) = \frac{3}{4}x + \frac{5}{4x^3},$$

$x_0 = 1$, and define

$$x_n = f(x_{n-1}) \text{ for } n = 1, 2, 3, \dots$$

Give the number x_n that gets close to when n is large.

- a. 1.4951
- b. 1.4953
- c. 1.4955
- d. 1.4957
- e. 1.4959
- f. None of these.

13. Find the area of the region that lies within the circle $x^2 + 4x + y^2 + 17y = 63$ and above the line $10x + 13y = -14$.
- 61.8545
 - 61.8541
 - 61.8543
 - 61.8539
 - 61.8537
 - None of these.
14. Find the area of the region that lies within the circle $x^2 + 4x + y^2 + 17y = 63$ and below the line $10x + 13y = -14$.
- 375.6126
 - 375.6128
 - 375.6122
 - 375.6124
 - 375.6130
 - None of these.
15. Find the shortest distance from the circle $x^2 + 31x + y^2 + 13y = 42$ to the point (17,31).
- 31.6099
 - 31.6095
 - 31.6097
 - 31.6101
 - 31.6103
 - None of these.
16. Find the largest distance from the circle $x^2 + 31x + y^2 + 13y = 42$ to the point (17,31).
- 67.6376
 - 67.6372
 - 67.6370
 - 67.6374
 - 67.6368
 - None of these.
17. Give the area of the triangle with vertices given by the intersections of the lines $12x - 13y = 16$, $4x + 3y = -12$ and $x + 3y = 16$.
- 59.0031
 - 59.0035
 - 59.0039
 - 59.0037
 - 59.0033
 - None of these.

18. Give the number of ways that numbers a , b and c can be chosen from the set

$$\{-10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

so that $a > 0$ and $ab - c^2 > 0$.

- a. 985
- b. 983
- c. 979
- d. 988
- e. 986
- f. None of these.

19. Give the number of ways that numbers a , b and c can be chosen from the set

$$\{-10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

so that $a < 0$ and $ab - c^2 > 0$.

- a. 988
- b. 979
- c. 983
- d. 986
- e. 965
- f. None of these.

20. Give the value of

$$\sum_{n=1}^{500} \frac{(-1)^{n+1}}{2n+3}.$$

- a. 0.1184
- b. 0.1182
- c. 0.1186
- d. 0.1190
- e. 0.1188
- f. None of these.

21. Find the shortest distance from the point $(2,1)$ to the set

$$S = \{(\cos(n), \sin(n)) : n = 1, 2, 3, \dots, 100 \text{ radians}\}.$$

- a. 1.2362
- b. 1.2360
- c. 1.2368
- d. 1.2364
- e. 1.2366
- f. None of these.

22. Find the largest distance from the point $(2,1)$ to the set

$$S = \{(\cos(n), \sin(n)) : n = 1, 2, 3, \dots, 100 \text{ radians}\}.$$

- a. 3.2359
- b. 3.2363
- c. 3.2367
- d. 3.2361
- e. 3.2365
- f. None of these.

23. Find the sum of the lengths of the line segments from the point $(2,1)$ to the set

$$S = \{(\cos(n), \sin(n)) : n = 1, 2, 3, \dots, 100 \text{ radians}\}.$$

- a. 235.5299
- b. 235.5303
- c. 235.5301
- d. 235.5305
- e. 235.5997
- f. None of these.

24. $S = \{13, 37, 91\}$. A sequence x_1, x_2, x_3, \dots is created so that x_1 is chosen at random from S , and then for $n = 2, 3, \dots$ a number a_n is chosen at random from S and $x_n = a_n + x_{n-1}$. Give the sum of the natural numbers ≤ 200 that will never be chosen in a sequence of this type.

- a. 13416
- b. 13417
- c. 13418
- d. 13419
- e. 13420
- f. None of these.

25. $S = \{13, 37, 91\}$. A sequence x_1, x_2, x_3, \dots is created so that x_1 is chosen at random from S , and then for $n = 2, 3, \dots$ a number a_n is chosen at random from S and $x_n = a_n + x_{n-1}$. Give the largest natural number ≤ 200 that will never be chosen in a sequence of this type.

- a. 194
- b. 198
- c. 197
- d. 199
- e. 195
- f. None of these.

26. Find the sum of all the solutions to the equation

$$100 \cos(x) + \frac{1}{15}x^3 + \frac{1}{2}x - 1 = 0.$$

- a. 10.7525
- b. 10.7527
- c. 10.7523
- d. 10.7521
- e. 10.7519
- f. None of these.

27. $(x_0, y_0) = (0,0)$, and for each $n = 1, 2, 3, \dots, 200$ the point (x_n, y_n) is given using the formulas

$$x_n = x_{n-1} + \cos(n^2), \quad y_n = y_{n-1} + \sin(n^2)$$

where the values n are given in radians. Give $x_{200} + y_{200}$.

- a. 11.6973
- b. 11.6981
- c. 11.6981
- d. 11.6975
- e. 11.6979
- f. None of these.

28. $(x_0, y_0) = (0,0)$, and for each $n = 1, 2, 3, \dots, 200$ the point (x_n, y_n) is given using the formulas

$$x_n = x_{n-1} + \cos(n^2), \quad y_n = y_{n-1} + \sin(n^2)$$

where the numbers n are given in radians. What is the sum of the lengths of the line segments connecting (x_{n-1}, y_{n-1}) to (x_n, y_n) for $n = 1, 2, \dots, 200$?

- a. 198
- b. 199
- c. 201
- d. 200
- e. 197
- f. None of these.

29. Let $a_n = 1 + \frac{1}{n}$, $b_n = 2 - \frac{1}{n}$ and T_n be a triangle with vertices $(0,4)$, $(-b_n, 0)$ and $(a_n, 0)$ for $n = 1, 2, \dots, 100$. Give the sum of the areas of the triangles T_n for $n = 1, 2, \dots, 100$.

- a. 300
- b. 150
- c. 550
- d. 450
- e. 750
- f. None of these.

30. Let $a_n = 1 + \frac{1}{n}$, $b_n = 2 - \frac{1}{n}$ and T_n be a triangle with vertices $(0,4)$, $(-b_n, 0)$ and $(a_n, 0)$ for $n = 1, 2, \dots, 100$. Give the sum of the circumferences of the triangles T_n for $n = 1, 2, \dots, 100$.

- a. 1158.7969
- b. 1158.7967
- c. 1158.7965
- d. 1158.7963
- e. 1158.7961
- f. None of these.

31. Determine the set of all points (x, y) that solve the system

$$\begin{cases} x + y = 3t \\ x - 2y = 6 \end{cases}$$

for some real number t . What is the distance from this set to the point $(-3, -2)$?

- a. 2.2363
- b. 2.2359
- c. 2.2365
- d. 2.2369
- e. 2.2361
- f. None of these.

32. Determine the set of all points (x, y) that solve the system

$$\begin{cases} x + y = 3t \\ x - 2y = 6 \end{cases}$$

for some real number t . What is the sum of the coordinates of the point on this set closest to $(-3, -2)$?

- a. -5
- b. -6
- c. -7
- d. -5.5
- e. -6.5
- f. None of these.

33. Determine the number of points of intersection of the line $2x - 11y = 1$ and the graph of

$$y = 100 \cos(x) + \frac{1}{15}x^3 + \frac{1}{2}x - 1.$$

- a. 8
- b. 7
- c. 6
- d. 9
- e. 11
- f. None of these.

34. Give the smallest natural number n so that

$$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} > 9.1851$$

- a. 5465
- b. 5446
- c. 5451
- d. 5483
- e. 5474
- f. None of these.

35. Give the largest natural number n so that

$$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} < 9.4641$$

- a. 7215
- b. 7234
- c. 7255
- d. 7265
- e. 7213
- f. None of these.

36. Give the number of points of intersection of the parametric curve

$$\left(\frac{10}{t+1} \cos(t), \frac{10}{t+2} \sin(t+2) \right) \text{ for } 0 \leq t \leq 30 \text{ radians}$$

with the axes in the xy -plane.

- a. 19
- b. 18
- c. 20
- d. 17
- e. 22
- f. None of these.

37. Give the largest value of t at which the parametric curve

$$\left(\frac{10}{t+1} \cos(t), \frac{10}{t+2} \sin(t+2) \right) \text{ for } 0 \leq t \leq 30 \text{ radians}$$

intersects an axis in the xy -plane.

- a. 29.8451
- b. 29.8449
- c. 29.8447
- d. 29.8445
- e. 29.8443
- f. None of these.

38. A number is represented in base 2 as 1100110011001100110011. Write this number in base 10.

- a. 3355446
- b. 3355449
- c. 3355452
- d. 3355443
- e. 3355455
- f. None of these.

39. Write the base 10 number 9819 in base 2 and then give the sum the digits in the base 2 representation.

- a. 5
- b. 9
- c. 11
- d. 10
- e. 8
- f. None of these.

40. Determine the number solutions to the system

$$\begin{cases} b + 2c + d + 3e = 80 \\ 3b + c + 4d + e = 110 \end{cases}$$

where each of b, c, d and e are positive integers.

- a. 69
- b. 71
- c. 57
- d. 59
- e. 83
- f. None of these.

41. Give the average of the sum of the coordinates of (b, c, d, e) of the solutions to the system

$$\begin{cases} b + 2c + d + 3e = 80 \\ 3b + c + 4d + e = 110 \end{cases}$$

where each of b, c, d and e are positive integers.

- a. 48.1737
- b. 48.1733
- c. 48.1739
- d. 48.1731
- e. 48.1735
- f. None of these.