

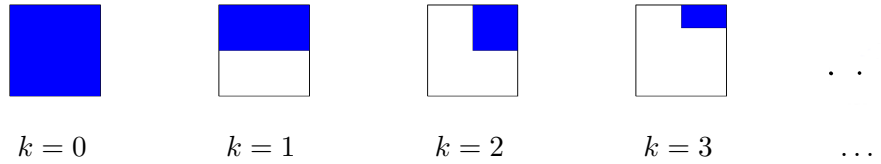
Algebra 1 Exam

University of Houston Math Contest 2025

1. Two trains are following each other on a track. Initially there is 50 miles between the back of the first train and the front of the second train. The train behind has a constant speed of 50 miles per hour and the train in front has a constant speed of 30 miles per hour. How long does it take for the front of the train behind to catch the back of the train in front?
 - (a) 130 mins
 - (b) 170 mins
 - (c) 2 hours
 - (d) 2 and a half hours
 - (e) 220 mins
 - (f) None of the above.
2. If the product of three consecutive integers is 120, what is the average value of these integers?
 - (a) 5
 - (b) 40
 - (c) 4.5
 - (d) 6
 - (e) 4
 - (f) None of the above.
3. The height of a rectangle is 4 inches more than its width. If the perimeter of the rectangle is 60 inches, what is the length of the longest side of the rectangle?
 - (a) 12 inches
 - (b) 16 inches
 - (c) 13 inches
 - (d) 15 inches
 - (e) 17 inches
 - (f) None of the above.

4. How many distinct real numbers solve the equation $0 = (x - 1)(-1 + x)(x^3 + 1)$?
- (a) 1
 - (b) 2
 - (c) 3
 - (d) 4
 - (e) 5
 - (f) None of the above.
5. Consider two lines in the xy plane. The first line passes through the points $(0, 2)$ and $(3, 3)$, and the second line passes through the points $(-2, -2)$ and $(2, 0)$. Which of the following statements is true?
- (a) The first line has a steeper slope than the second one.
 - (b) The second line has a negative slope.
 - (c) The two lines are parallel to each other.
 - (d) The lines intersect at a point with negative x coordinate.
 - (e) The lines intersect at a point with a y coordinate that lies in the interval $[0, 3]$.
 - (f) None of the above.
6. Find the values a and b so that the point (a, b) on the line $y = 2x$ is closest to the point $(0, 1)$.
- (a) $a = 1/5, b = 2/5$
 - (b) $a = 0, b = 0$
 - (c) $a = 1/3, b = 1/3$
 - (d) $a = 2/5, b = 4/5$
 - (e) $a = 1/3, b = 2/3$
 - (f) None of the above.
7. Rank the following numbers in ascending order: $\sqrt{12}$, $2\sqrt{2}$, $\sqrt{2} + \sqrt{3}$ and $1 + \sqrt{3}$.
- (a) $2\sqrt{2} < 1 + \sqrt{3} < \sqrt{2} + \sqrt{3} < \sqrt{12}$
 - (b) $2\sqrt{2} < 1 + \sqrt{3} < \sqrt{12} < \sqrt{2} + \sqrt{3}$
 - (c) $1 + \sqrt{3} < 2\sqrt{2} < \sqrt{2} + \sqrt{3} < \sqrt{12}$
 - (d) $\sqrt{12} < 1 + \sqrt{3} < 2\sqrt{2} < \sqrt{2} + \sqrt{3}$
 - (e) $\sqrt{12} < \sqrt{2} + \sqrt{3} < 2\sqrt{2} < 1 + \sqrt{3}$
 - (f) None of the above.

8. Start from a unit square and cut it in half, alternating in the vertical and horizontal directions as shown below. Select the statement that is false.



- (a) width/height of the blue rectangle is always equal to either 1 or 2.
 - (b) The area of the blue rectangle after k iterations is equal to $1/2^k$.
 - (c) The blue rectangle is a square whenever k is even.
 - (d) The perimeter of the blue rectangle is divided by 2 from iteration k to iteration $k + 1$.
 - (e) The area of the blue rectangle is divided by 2 from iteration k to iteration $k + 1$.
 - (f) None of the above.
9. Find the equation of the line that passes through the point $(3, -1)$ and is perpendicular to the line $2x - y = 1$.
- (a) $2x - y = -1$
 - (b) $x + 2y = 2$
 - (c) $x + 3y = 0$
 - (d) $-x - 2y = 1$
 - (e) $y = -1$
 - (f) None of the above.
10. What is the greatest common divisor of the numbers 252 and 280?
- (a) 2
 - (b) 4
 - (c) 12
 - (d) 14
 - (e) 28
 - (f) None of the above.

11. Michaelangelo, Leonardo, Raphael, Donatello and Splinter are sitting next to an 8×8 chess board they will use to decide who will pay for pizza. Michaelangelo starts by putting a grain of rice on one of the 64 squares, then Leonardo picks two grains and puts them on two new squares, Raphael proceeds with 3 grains and puts them on three additional squares, Donatello does the same with 4 grains, Splinter with 5, and the game continues looping back to Michaelangelo, each person having to use one more grain of rice than the previous one and selecting unused squares. The first person who can't place all of their grains on the board has to buy the pizza. Who buys the pizza?
- (a) Michaelangelo
 - (b) Leonardo
 - (c) Raphael
 - (d) Donatello
 - (e) Splinter
 - (f) None of the above.
12. Given the function $h(x) = x^2 + x + 3$. Evaluate and simplify $h(x - 3)$.
- (a) $x^2 - 7x + 15$
 - (b) $x^2 - 6x + 9$
 - (c) $x^2 - 5x + 9$
 - (d) $2x - 3$
 - (e) $x^2 + 5x + 9$
 - (f) None of the above.
13. 3 fair dice with 6 sides are tossed together. What is the probability that at least two of them have the same number?
- (a) $1/6$
 - (b) $1/3$
 - (c) $4/9$
 - (d) $5/9$
 - (e) $2/3$
 - (f) None of the above.

14. Simplify the expression:

$$\frac{\sqrt{a^3} \sqrt{b} c}{a^2 b^{-1/4} \sqrt{ac}}.$$

- (a) $\frac{1}{a}$
- (b) $\frac{\sqrt{b}}{a}$
- (c) $\sqrt{\frac{b}{a}}$
- (d) $\frac{\sqrt{bc}}{a}$
- (e) $\sqrt{ab}$
- (f) None of the above.

15. Two periodic phenomena occur every 15 and 39 years, respectively. Both phenomena occur in 2025. When is the next year they both occur?

- (a) 2140
- (b) 2164
- (c) 2220
- (d) 2270
- (e) 2310
- (f) None of the above.

16. Evaluate and simplify the fraction

$$\frac{1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17}{1 + 2 + 4 + 8 + 16 + 32}$$

- (a) $3/2$
- (b) $5/7$
- (c) $9/7$
- (d) $81/31$
- (e) $64/63$
- (f) None of the above.

17. Iodine 131 is a radioactive isotope that loses about 10% of its mass every day. After how many days does a given amount of iodine 131 reduce to less than half of its initial mass?

(a) 8
(b) 6
(c) 9
(d) 11
(e) 7
(f) None of the above.

18. Consider the three functions

$$f_1(x) = -x^2 + 2x + 1, \quad f_2(x) = x^2 + \frac{2}{3}, \quad \text{and} \quad f_3(x) = -2x^2 + 4x.$$

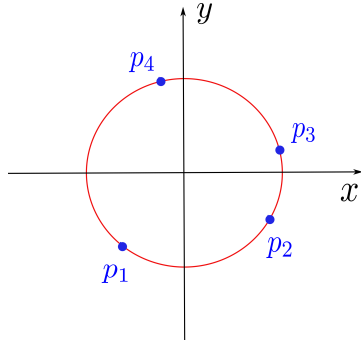
M_1 and M_3 are the maximum values of the first and third functions, and M_2 is the minimum value of the second function. Find the correct statement below.

(a) $M_1 < 1/M_2 < \sqrt{M_3}$
(b) $\sqrt{M_3} < 1/M_2 < M_1$
(c) $M_1 < \sqrt{M_3} < 1/M_2$
(d) $1/M_2 < M_1 < \sqrt{M_3}$
(e) $1/M_2 < \sqrt{M_3} < M_1$
(f) None of the above.

19. Julie has a stack of an unknown number of cards that were taken from a deck of 52 cards. When dealing those into piles of 3 cards, she is eventually left with a single card. When dealing them into piles of 4 cards, she ends up with 2 cards. When dealing them into piles of 5 cards, she ends up with 1 card. How many cards were in the original stack?

(a) 16
(b) 22
(c) 26
(d) 46
(e) There is not enough information to answer the question.
(f) None of the above.

20. The figure below shows a circle in the xy plane centered at the origin, on which the four points p_1, p_2, p_3, p_4 are marked in blue. Let s_1, s_2, s_3, s_4 be the slopes of the tangent lines to the circle at those respective points. Which of the following is true?



- (a) All slopes s_1, s_2, s_3, s_4 are positive.
- (b) $s_3 < s_1 < s_4 < s_2$
- (c) $s_3 < s_4 < s_1 < s_2$
- (d) $s_1 < s_2 < s_3 < s_4$
- (e) The slope s_3 is larger than the slope s_1 .
- (f) None of the above.

21. Give the number of real solution(s) (x, y) for the system of equations

$$\begin{cases} x^2 + 6x + 9 = 0 \\ y^2 + xy + 3 = 0 \end{cases}$$

- (a) The system has no real solution.
- (b) 1 solution
- (c) 2 solutions
- (d) 3 solutions
- (e) 4 solutions
- (f) None of the above.

22. Consider an arithmetic progression $a_1, a_2, a_3, \dots$, where

$$a_1 + a_2 + a_3 + a_4 = 20 \quad \text{and} \quad a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 = 104.$$

Determine the value of the first term a_1 and the common difference d of the arithmetic progression.

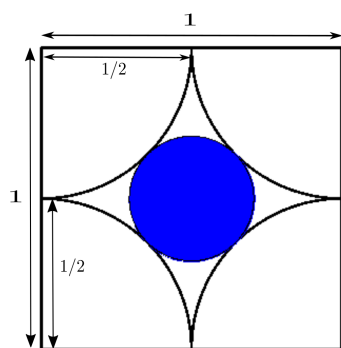
- (a) $a_1 = 0, d = 5$
- (b) $a_1 = 2, d = 2$
- (c) $a_1 = -1, d = 4$
- (d) $a_1 = -3, d = 4$
- (e) $a_1 = -4, d = 5$
- (f) None of the above.

23. Give the x -coordinate of the vertex of the parabola given by

$$y - x^2 = 4x - 4y.$$

- (a) 2
- (b) 0
- (c) 1
- (d) -1
- (e) -2
- (f) None of the above.

24. What is the area of the blue disk in the figure below?



- (a) $(\frac{3}{4} - \frac{\sqrt{2}}{2})\pi$
- (b) $\frac{\pi}{9}$
- (c) $\frac{\sqrt{2}}{2}\pi$
- (d) $1 - \frac{\pi}{4}$
- (e) $\frac{\pi}{8}$
- (f) None of the above.

25. Bill has placed \$100 in a financial fund that has a fixed yearly return rate of $R\%$. Although he forgot the rate, he learns that after 2 years he has \$121 in the fund. What is the yearly rate R ?

- (a) 2.1
- (b) 10
- (c) 11
- (d) 12.1
- (e) 21
- (f) None of the above.

26. The factorial of positive integer n is written $n!$, and the values are given by

$$1! = 1, \quad 2! = 1 \times 2 = 2, \quad 3! = 2! \times 3 = 6, \quad 4! = 3! \times 4 = 24, \quad \text{etc.}$$

Which of the following numbers is a perfect square?

(a) $\frac{27! \times 28!}{3}$

(b) $\frac{17! \times 18!}{3}$

(c) $\frac{14! \times 15!}{3}$

(d) $\frac{23! \times 24!}{3}$

(e) $\frac{26! \times 27!}{3}$

(f) None of the above.

27. Assume we are given the function $f(x) = 2x^2 + bx + c$ where b and c are unknown, but we are told that -2 and 3 are zeros of this function. Find the values of b and c .

(a) $b = 1$ and $c = -6$

(b) $b = -2$ and $c = 12$

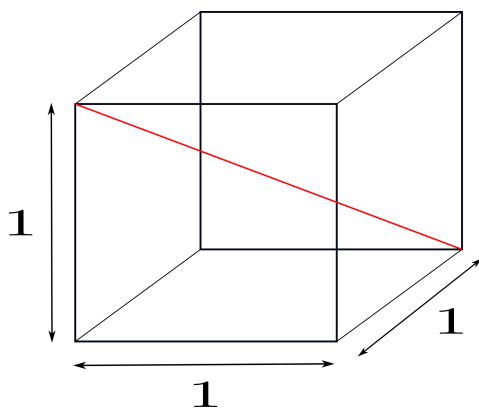
(c) $b = 2$ and $c = 12$

(d) $b = -2$ and $c = -12$

(e) $b = -2$ and $c = 3$

(f) None of the above.

28. What is the length of the diagonal of a 3D cube whose sides have length 1?



(a) 1

(b) $\sqrt{2}$

(c) $\sqrt{3}$

(d) 2

(e) $\sqrt{5}$

(f) None of the above.

29. What is the average slope of the three lines l_1 , l_2 and l_3 that pass through the origin, given that l_1 , l_2 and l_3 pass through $(2, 3)$, $(4, -6)$ and $(3, \sqrt{2})$, respectively?
- (a) $\frac{2}{3}$
 - (b) $\frac{3}{2}$
 - (c) $\frac{\sqrt{2}}{3}$
 - (d) $\frac{3}{\sqrt{2}}$
 - (e) $\frac{\sqrt{2}}{9}$
 - (f) None of the above
30. An indoor basketball court has several lights in the ceiling. One of the lights is h meters directly above the center of a free-throw line, and this light is s meters from midcourt. Midcourt is 5 meters from the free throw line. Find h given that $s + h = 25m$.
- (a) 10m
 - (b) 12m
 - (c) 12.5m
 - (d) 15m
 - (e) 25m
 - (f) None of the above.
31. Determine the solutions of the inequality $x^2 - 6x + 9 \geq 1$.
- (a) $2 \leq x \leq 4$
 - (b) $-4 \leq x \leq -2$
 - (c) $x \leq 2$ or $x \geq 4$
 - (d) $x \leq -4$ or $x \geq -2$
 - (e) There are no solutions.
 - (f) None of the above.

32. A deck of 52 cards contains cards showing the numbers

$$1, 2, 3, \dots, 13,$$

with each numbered card appearing 4 times in the deck. If two cards are taken at random from this deck, what is the probability that both cards have the same number?

- (a) $1/4$
 - (b) $3/13$
 - (c) $1/13$
 - (d) $1/17$
 - (e) $1/51$
 - (f) None of the above.
33. The sum of George's, Anne's and Jim's ages is 31. George is two years older than Anne. Moreover, three years ago, Jim was twice as old as Anne. How old is Anne?
- (a) 10
 - (b) 8
 - (c) 6
 - (d) 11
 - (e) 5
 - (f) None of the above.
34. Which value of x satisfies the equation:

$$1 - \frac{1}{x} = \frac{x^2}{1 + \frac{1}{x}}?$$

- (a) $x = -1$
- (b) $x = \sqrt{2}$
- (c) $x = \frac{1}{\sqrt{2}}$
- (d) $x = -\frac{1}{2}$
- (e) $x = 0$
- (f) None of the above.

35. We are given two bags of coins. The first bag contains 4 identical gold coins and the second bag contains 6 identical silver coins. When we weigh them, we find that the first bag is 2 oz heavier than the second bag. Then, if we switch one gold coin from the first bag with a silver coin from the second bag, the first bag becomes 2 oz lighter than the second bag. Select the statement that is true.
- (a) Each gold and silver coin weighs 4 oz.
 - (b) A gold coin weighs 6 oz and a silver one 4 oz.
 - (c) A gold coin weighs twice as much as a silver coin.
 - (d) A silver coin weighs 3 oz and a gold one 5 oz.
 - (e) A gold coin weighs 4 oz more than a silver coin.
 - (f) None of the above.
36. A and B are numbers. We know $A + B = 44$, and $(x + 4)$ is a factor of the polynomial $x^2 + Ax + B$. What is A ?
- (a) 6
 - (b) 13
 - (c) 20
 - (d) 12
 - (e) 32
 - (f) None of the above.
37. Sequence 1 has consecutive terms $100, 120, 140, 160, \dots$, and sequence 2 has consecutive terms $1, -2, 4, -8, \dots$. If you compare the n^{th} terms in each sequence (where $n = 1$ corresponds to the first term), what is the minimum value of n for which the n^{th} term in sequence 2 is larger than the n^{th} term in sequence 1?
- (a) It never happens.
 - (b) 8
 - (c) 9
 - (d) 10
 - (e) 11
 - (f) None of the above.

38. A medical trial involved treatments T1 and T2 that were tested on patients divided in two groups G1 and G2. Treatment T1 was administered to 9 patients of group G1, with success on 8 of them, and administered to 26 patients of group G2, with success on 19 of them. Treatment T2 was administered to 27 different patients from G1, with success on 23 of them, and it was administered to 8 different patients from G2, with success on 5 of them. Which of the following statements is false?
- (a) The two treatments T1 and T2 have each been used on a total of 35 patients.
 - (b) T1 has a better success rate than T2 on group G1.
 - (c) T1 has a better success rate than T2 on group G2.
 - (d) T1 has a better success rate than T2 when considering the two groups combined together.
 - (e) The two groups do not have the same number of patients.
 - (f) None of the above.
39. A two digit number is written without using the digit zero. When you reverse my digits and subtract 27 from the new number you get the original number. How many such numbers are there?
- (a) 2
 - (b) 3
 - (c) 4
 - (d) 5
 - (e) 6
 - (f) None of the above.
40. Find the distance between the points $(\sqrt{2}, 3)$ and $(3, \sqrt{2})$.
- (a) $3\sqrt{2} - 2$
 - (b) $2 - 3\sqrt{2}$
 - (c) 4
 - (d) $\sqrt{11}$
 - (e) $\sqrt{22}$
 - (f) None of the above.
41. A line passes through the point $(2, 1)$. Give the largest difference between values a and b so that if m is the slope of the line and $a < m < b$ then the line does not intersect the parabola $y = x^2 - 3x + 6$.
- (a) $4\sqrt{3}$
 - (b) $8\sqrt{3}$
 - (c) $5\sqrt{2}$
 - (d) $3\sqrt{2}$
 - (e) $4\sqrt{2}$
 - (f) None of the above