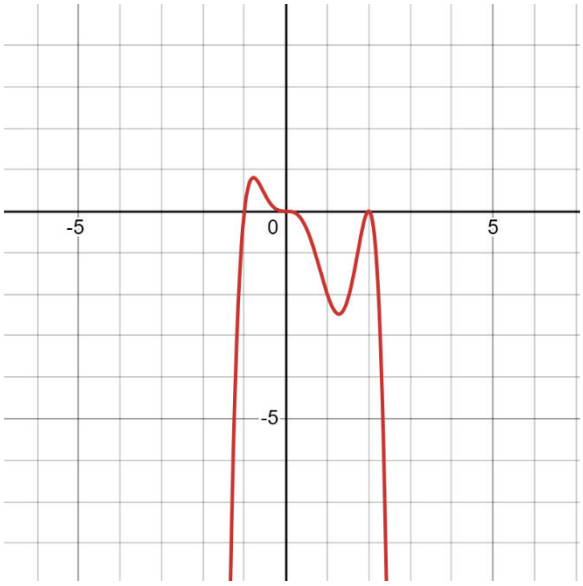
Algebra 2 Exam

University of Houston Math Contest 2025

1. Which of the following could be an equation for the graph $y = f(x)$ given below?

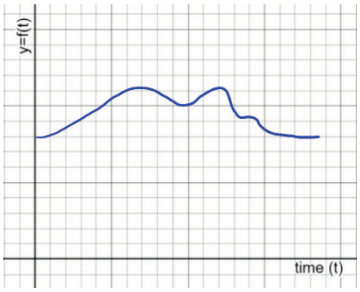


A. $f(x) = x^2\,(x-1)(x+2)$ B. $f(x) = -x^2\,(x-1)(x+2)$ C. $f(x) = x(x+1)(x-2)^2$

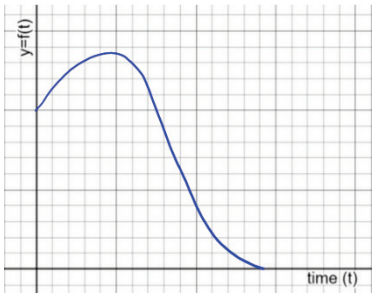
D. $f(x) = -x\,(x-1)(x-2)^2$ E. $f(x) = -x^3\,(x+1)^2(x-2)$ F. $f(x) = -x^3\,(x+1)(x-2)^2$

2. Which of the graphs below best illustrate the following situation?
The price of gas is rising over a period of time, but fortunately, by less each month.

A.



B.



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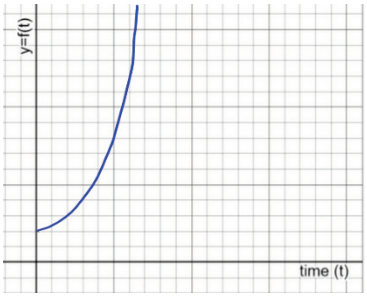
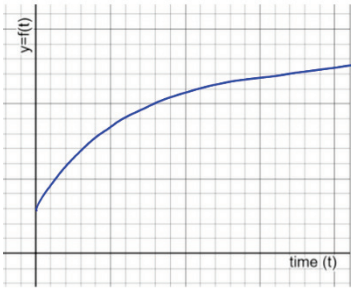
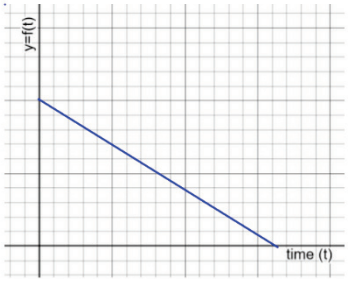


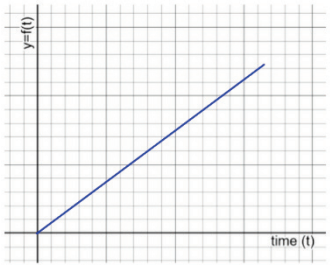
D.



E.



F.



3. State the domain for the given function: $f(x) = \frac{\sqrt{x^2-9}}{x^2+2x-15}$.

A. $[-3, \infty)$ B. $(-\infty, -3) \cup (3, \infty)$ C. $(-\infty, -5) \cup [-3,3) \cup (3, \infty)$

D. $(-\infty, -5) \cup (-5, -3] \cup (3, \infty)$ E. $(-5, -3]$ F. None of these

4. Find $f\left(g^{-1}(5)\right)$, if $f(x) = \frac{2}{x}$ and $g(x) = \log_2(x+3)$.

A. $-\frac{11}{5}$ B. $\frac{2}{29}$ C. $\frac{2}{3}$

D. $\sqrt[5]{4} - 3$ E. $4 - \log_2(5)$ F. Not possible.

5. Three friends go on vacation and their budget for housing, food and transportation cannot total more than \$5000. They have figured that the total for food and transportation must be less than the housing budget. With the restriction that the food budget is less than twice expected cost of transportation, which of the following systems can represent this silly set of restrictions?

A. $\begin{cases} H - F - T \leq 5000 \\ F + T < H \\ 2F < T \end{cases}$ B. $\begin{cases} H + F + T \leq 5000 \\ F - T < H \\ 2F < T \end{cases}$ C. $\begin{cases} H + F + T \leq 5000 \\ F + T < H \\ F < 2T \end{cases}$

D. $\begin{cases} H + T \leq 5000 + F \\ F < \frac{1}{2}T \\ H > F + T \end{cases}$ E. $\begin{cases} H - F - T \leq 5000 \\ F + T < H \\ 2F < T \end{cases}$ F. None of these

6. Let M represent the maximum value of the function $f(x) = b^x$ on the interval $[-1,2]$ and $m = \min\left(f(x)\right)$ represent the minimum value on the same interval.

What is the value of $M - m$, for any $0 < b < 1$?

A. b B. $1 - b$ C. $\frac{1}{b}$

D. $\frac{1}{b} - b^2$ E. $b^2 - \frac{1}{b}$ F. None of these

7. Given $f(x) = 2\ln(x)$ and $g(x) = e^{7x}$, solve the following equation for x: $f\left(g(x)\right) = g\left(f(x)\right)$.

A. $\{0,14\}$ B. $\{14\}$ C. $\{0\}$

D. $\{\sqrt[13]{14}\}$ E. $\{0, \sqrt[13]{14}\}$ F. None of these

8. Natalie and Sachin are working on an art project together. If Natalie can complete the project on her own in 5 hours, how many hours does it take Sachin to complete the project on his own if we know that together they can complete the project in 3 hours?

A. $\frac{3}{7}$ B. $\frac{5}{7}$ C. $\frac{7}{2}$

D. $\frac{15}{2}$ E. 15 F. None of these

9. Solve the following inequality for x: $\frac{x^2-x}{x-3} > 0$.

A. $(0,\infty)$ B. $(-\infty,0) \cup (1,\infty)$ C. $(0,1) \cup (3,\infty)$

D. $(-\infty,0) \cup (1,3) \cup (3,\infty)$ E. $\left(-\sqrt{3},\sqrt{3}\right)$ F. None of these

10. If $f(-x) = -f(x)$ for all x in the domain of f. Find $f(-2)$ if $f^{-1}(7) = 2$.

A. 2 B. -2 C. 7

D. -7 E. f is not invertible. F. None of these

11. Which of the following is equivalent to $\frac{1}{2}\log_2(12)$?

A. $\log_2(6)$ B. $1 + \frac{1}{2}\log_2(3)$ C. 3

D. $\sqrt{6}$ E. $2 + \log_2(3)$ F. None of these

12. Let f be a function that can be written in the form $f(x) = ax^2 + bx + c$, where a, b, c are any real number and $y = f(x)$ has a line of symmetry, $x = 7$. If $a > 0$, which of the following statements must be true:

i. $f(7) = c$

ii. $b = 14a$

iii. $b = -14a$

iv. The maximum value of $f(x)$ is $f(7)$.

v. The minimum value of $f(x)$ is $f(7)$.

A. only i, ii, iv are true B. only i, ii, v are true C. only ii, iv are true

D. only ii, v are true E. only iii, iv are true F. only iii, v are true

13. If the graph of $f(x) = 3x - 2$ intersects the graph of $g(x) = ax^2 + bx + 5$ at $x = -1$ and $x = 4$, find the value of a.

A. $\frac{15}{4}$ B. $\frac{33}{4}$ C. $-\frac{7}{4}$

D. $-\frac{9}{4}$ E. $-\frac{35}{12}$ F. None of these

14. If the directrix of a parabola is $y = 7$ and the focus of the parabola is $(3, c)$, where c is any real number. Which of the following are possible values of (h, k), the vertex of this parabola?

A. $(3, c - 7)$ B. $(3, c + 7)$ C. $\left(3, \frac{7-c}{2}\right)$

D. $(5, c)$ E. $\left(3, \frac{c+7}{2}\right)$ F. None of these

15. Let $f(x) = \frac{x^3-7x+1}{x^2+2}$, where $f(x)$ can be written as $f(x) = \frac{N(x)}{D(x)} + R(x)$, with both $N(x)$ and $D(x)$ being polynomials such that the degree of $D(x)$ is less than the degree of $N(x)$. What is $R(x)$?

A. $-5x + 1$ B. $\frac{-9x+1}{x^2+2}$ C. $-9x + 1$

D. $\frac{-5x+1}{x^2+2}$ E. $\frac{-9x-1}{x^2+2}$ F. None of these

16. Find the domain for the function h, if $h(x) = \frac{f^{-1}(x)}{(x^2+9)(x-7)}$ and $f(x) = e^{x^2} - 5$.

A. $(-\infty, -3) \cup (-3, 3) \cup (3, 7)$ B. $(-\infty, -5) \cup (-5, 7) \cup [7, \infty)$ C. $(-5, 7) \cup (7, \infty)$

D. $[-4, 7) \cup (7, \infty)$ E. $[-5, 7) \cup (7, \infty)$ F. None of these

17. The graph $y = f(x)$ contains the point (a, b). Which of the following is a point on the graph of $y = f(3x - 2) - 7$?

A. $(3a + 2, b - 7)$ B. $\left(\frac{a+2}{3}, b + 7\right)$ C. $\left(\frac{a+2}{3}, b - 7\right)$

D. $\left(\frac{a}{3} + 2, b + 7\right)$ E. $\left(\frac{2a}{3}, b - 7\right)$ F. None of these

18. Let $f(x) = 2x^2 - 4x - 6$. Write an equation for a line that contains the largest root of f and the y-intercept of $y = f(x)$.

A. $2x - 3y = 6$ B. $2x - y = 6$ C. $y - 2x = -3$

D. $y = 3x + 6$ E. $y = 3x + 3$ F. None of these

19. Let $h(x) = \frac{2}{x+7} - 3$. Decompose h so that $h(x) = f\big(g(x)\big)$. What are possible options for f and g?

i. $f(x) = x + 7, \quad g(x) = \frac{2}{x} - 3$

ii. $f(x) = \frac{2}{x} - 3, \quad g(x) = x + 7$

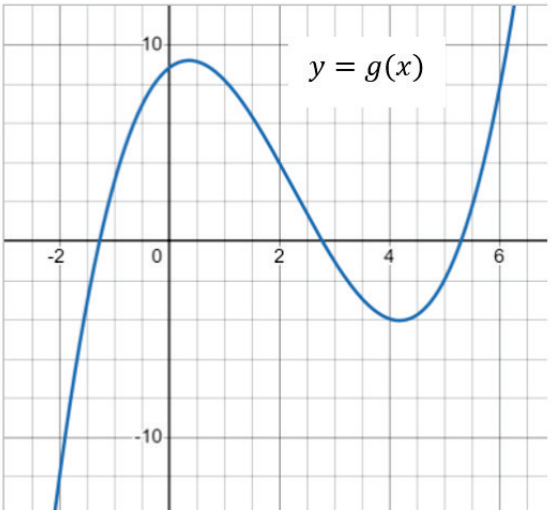
iii. $f(x) = x - 3, \quad g(x) = \frac{2}{x+7}$

iv. $f(x) = \frac{2}{x+7}, \quad g(x) = x - 3$

A. all of $i, ii, iii, and\ iv$ B. none of $i, ii, iii,$ or iv C. only $i\ and\ ii$

D. only $i\ and\ iv$ E. only $ii\ and\ iii$ F. only $iii\ and\ iv$

20. The graph of $y = g(x)$ and a table of values for the function f are given below. Use both the graph and the table to evaluate $g\big(f^{-1}(3)\big)$.



x	$f(x)$
-1	-3
1	1
3	2
6	3
7	5

A. -3 B. -2 C. 8

D. 2 E. 4 F. None of these

21. Let $a_1 = 5$ and $a_{n+1} = \frac{a_n}{2}$. Find a_4 and an explicit formula for the n^{th} term, a_n.

A. $a_4 = \frac{5}{4}, \quad a_n = \left(\frac{5}{2}\right)^n$ B. $a_4 = \frac{5}{4}, \quad a_n = \left(\frac{5}{4}\right)^n$ C. $a_4 = 1, \ a_n = \frac{n}{2} - 1$

D. $a_4 = \frac{5}{8}, \quad a_n = 5\left(\frac{1}{2}\right)^{(n)}$ E. $a_4 = \frac{5}{8}, \quad a_n = 5\left(\frac{1}{2}\right)^{(n-1)}$ F. None of these

22. Let R be the region of all points, (x, y) that satisfy the following inequalities:

$$\begin{cases} y > \frac{1}{5}x + 1 \\ x - y > 3 \end{cases}$$

If (a, b) is a point in R where a and b are both integers, find the minimum value of $a + b$.

A. 11 B. 7 C. 8

D. 9 E. 10 F. None of these

23. If $3x - y = 12$, find the value of

$$\frac{9x - 3y}{5} + \log_7\left(\frac{1}{49}\right)$$

A. $-\frac{6}{5}$ B. $-\frac{1}{5}$ C. $\frac{1}{5}$

D. $\frac{26}{5}$ E. $\frac{77}{10}$ F. None of these

24. For some real value, b, the graphs of the equation $y = -3|x + 1| + 2|x - 5|$ and the line $y = b$ intersect at exactly one point. What is the value of b?

A. -18 B. -13 C. -1

D. 7 E. 12 F. None of these

25. Solve for x:

$$\log_8(16 \log_x(3)) = 2$$

A. $-\sqrt[4]{3},\ \sqrt[4]{3}$ B. $-\sqrt{3},\ \sqrt{3}$ C. $\sqrt{3}$

D. $\sqrt[4]{3}$ E. 3 F. None of these

26. An equilateral triangle has side length s. Find the sum of its area and $\frac{1}{2}$ its perimeter.

A. $\frac{12s+\sqrt{3}s^2}{4}$ B. $\frac{6s+\sqrt{3}s^2}{4}$ C. $\frac{6s+s^2}{4}$

D. $\frac{s^2+2s^3}{2}$ E. $\frac{6s+s^2}{2}$ F. None of these

27. Find $B - 4A$ if $\frac{A}{x+1} + \frac{B}{(x-4)} = \frac{2x-3}{x^2-3x-4}$.

A. -3 B. 1 C. 2

D. 3 E. 4 F. None of these

28. Given that 3 and $2 - 5i$ are two of the roots of the polynomial, $f(x) = x^3 + ax^2 + bx + c$. Find the sum of a, b and c.

A. -87 B. -53 C. -22

D. 35 E. 41 F. None of these

29. If $4x + y = 8$ and $20x^2 - 7xy - 3y^2 = 40$, find the value of $15x - 9y$.

A. 5 B. 10 C. 15

D. 20 E. 25 F. None of these

30. Let (x, y) be the solution to the following system: $\begin{cases} \frac{\sqrt[4]{9}\cdot\sqrt{9^x}}{243} = 27^y \\ \frac{64^x}{4^y} = 2. \end{cases}$

Find the value of y.

A. $-\frac{11}{7}$ B. $-\frac{13}{8}$ C. $\frac{7}{4}$

D. $-\frac{23}{7}$ E. $\frac{11}{8}$ F. None of these

31. Given $f(x) = \frac{2-x}{x+4}$. Solve $f^{-1}(e^x) = 1$.

A. -1 B. 2 C. $\ln(5)$

D. $3^{\ln(5)}$ E. $-\ln(5)$ F. None of these

32. The intensity, I, of light from a lamp varies inversely with the square of the distance, d, from the lamp. If the intensity is 50 lux at a distance of 4 meters, what is the intensity at a distance of 16 meters?

A. $\frac{25}{4}$ B. $\frac{25}{8}$ C. $\frac{25}{16}$

D. 400 E. 800 F. None of these

33. Find the sum of the solutions to this equation: $\left(\frac{1}{25}\right)^{2x^2-2x-1} = 5^{\,2x^2+5x-10}$.

A. $-\frac{1}{12}$ B. $-\frac{1}{6}$ C. $-\frac{11}{6}$

D. $\frac{5}{12}$ E. $\frac{11}{12}$ F. None of these

34. Suppose that

$$\begin{bmatrix} 2t-3y & 10 & 4 \\ -5 & x+t & 8 \\ 0 & 1 & z-p+y \end{bmatrix} = \begin{bmatrix} 25 & 5x-2z & z+2y \\ -5 & 14 & 8 \\ 0 & 1 & 14 \end{bmatrix}$$

Find the value of $2t + 5x - 3y - 2z$.

A. 37 B. 17 C. 75

D. 35 E. 24 F. None of these

35. A pair of mischievous bunnies, Mr. Fluff and Mrs. Fluff, are loose in a magical garden. They have a habit of doubling their population every month. If they start with just the two of them, how many bunnies will be hopping around the garden after a year?

A. 2^{12} B. 2^{13} C. 12^2

D. $2 \cdot 12^2$ E. $2 \cdot 12$ F. None of these

36. Let $(a, f(a))$ be the coordinate to the point corresponding to the minimum value of $f(x)$and let $(b, g(b))$ be the coordinate to the point corresponding to the maximum value of $g(x)$. Find the sum of $f(a)$and $g(b)$ if $f(x) = -x^2 - 4x + 4$ and $g(x) = 3|x^2 - 4x - 5|$, if both functions have the domain $\{x: -1 \leq x \leq 6\}$.

A. $f(-2) + g(-1)$ B. $f(-1) + g(-1)$ C. $f(6) + g(6)$

D. $f(6) + g(2)$ E. $f(6) + g(5)$ F. None of these

37. Solve for x: $25^x + 21 \cdot 5^{x-1} = 4$.

A. $\left\{-5, \frac{4}{5}\right\}$ B. $\{\log_5(4) \ \ -1\}$ C. $\left\{\log_5\left(\frac{4}{5}\right), 5\right\}$

D. $\{-\ln(5)\}$ E. $\{\ln(4) - \ln(5)\}$ F. None of these

38. Let $f(x) = \frac{7x+x^2}{x^2-2x}$, where $y = h$ is the horizontal asymptote of the graph of $y = f(x)$, and $x = a$ is the vertical asymptote. Find:

$$\log_{\left(\frac{1}{3}\right)}(1 + h \cdot a) - \sqrt{(8h + 2a)^3}$$

A. $-7 - 24\sqrt{3}$ B. $-1 - 24\sqrt{3}$ C. $1 - 24\sqrt{3}$

D. $7 - 24\sqrt{3}$ E. $-16\sqrt{2}$ F. None of these