

Calculation Exam

University of Houston Mathematics Contest 2025

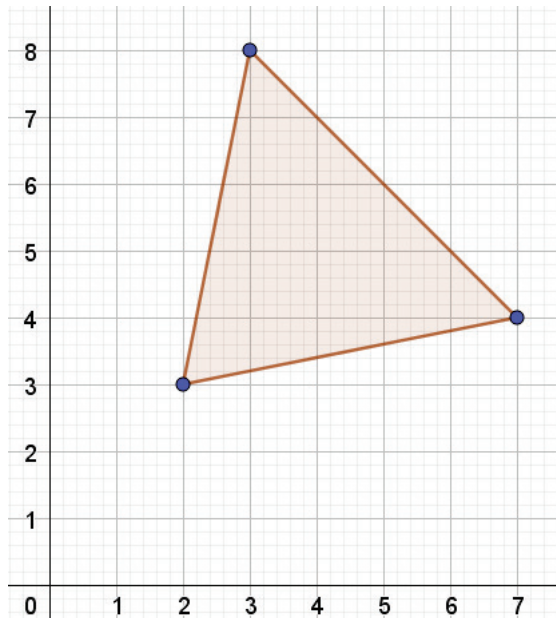
Directions: Select the answer that is closest to the correct answer.

1. S is the set containing the 127 smallest natural numbers that are divisible by 3. Give the sum of the elements in S .
 - a. 24489
 - b. 24381
 - c. 24184
 - d. 24288
 - e. 24384
 - f. 24487

2. Find the sum of the unique positive integer divisors of 12345678.
 - a. 27319971
 - b. 27319954
 - c. 27319968
 - d. 27319972
 - e. 27319960
 - f. 27319966

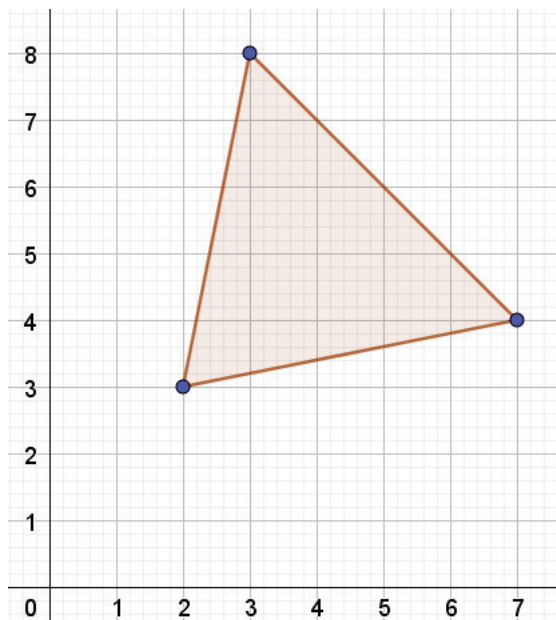
3. Give the average of the squares of the first 100 natural numbers.
 - a. 3681.5
 - b. 3522.5
 - c. 3426.5
 - d. 3244.5
 - e. 3371.5
 - f. 3383.5

4. Give the area of the triangle shown below.



- a. 14
- b. 13.5
- c. 13
- d. 12.5
- e. 12
- f. 11.5

5. The triangle shown below is divided into two pieces by the line $y = x$. Give the area of the smaller of the two pieces.



- a. 3.375
- b. 3.425
- c. 3.525
- d. 3.325
- e. 3.675
- f. 3.475

6. A sequence is generated by setting $x_1 = 1$,

$$x_2 = \frac{1 + x_1}{2}$$

$$x_3 = \frac{1 + x_1 - x_2}{3}$$

$$x_4 = \frac{1 + x_1 - x_2 + x_3}{4}$$

$$x_5 = \frac{1 + x_1 - x_2 + x_3 - x_4}{5}$$

$\vdots$

Give the value of x_{100} .

- a. 0.012684235
 - b. 0.010101010
 - c. 0.012186235
 - d. 0.010385235
 - e. 0.012387235
 - f. 0.010489235
7. S is the set containing the smallest 127 natural numbers that are not divisible by any of 2, 3, 5 or 7. What is the largest number in S ?
- a. 551
 - b. 557
 - c. 599
 - d. 613
 - e. 631
 - f. 643

8. S is the set containing the smallest 127 natural numbers that are not divisible by any of 2, 3, 5 or 7. Give the sum of the numbers that are in S .

- a. 33451
- b. 36453
- c. 37822
- d. 34671
- e. 34017
- f. 35273

9. Give the sum of the coordinates of the center of the circle given by

$$3x^2 - 7x + 3y^2 + 13y = 141.$$

- a. -2
- b. 3
- c. 4
- d. -1
- e. 1
- f. 2

10. Give the radius of the circle given by

$$3x^2 - 7x + 3y^2 + 13y = 141.$$

- a. 7.283924461
- b. 7.283921361
- c. 7.283923751
- d. 7.283927721
- e. 7.283926511
- f. 7.283927431

11. Give the area of the intersection of the circle given by

$$3x^2 - 7x + 3y^2 + 13y = 141$$

and the circle of radius 7 centered at (8,7).

- a. 15.48261332
 - b. 15.97633621
 - c. 15.48732115
 - d. 16.62185646
 - e. 16.31244421
 - f. 15.48652334
12. The line $y = x$ divides the circle of radius 7 centered at (8,7) into two pieces. Give the area of the smallest piece.

- a. 67.08638682
- b. 53.89623312
- c. 64.14617736
- d. 68.42556671
- e. 59.33472665
- f. 63.28816774

13. Give the absolute value of the difference of the coordinates of the point of intersection of the lines given by

$$27.6x - 341.5y = 131.2$$

and

$$-32.45x + 17.53y = 756.$$

- a. 22.25613216
- b. 22.20742534
- c. 21.42763113
- d. 22.48377512
- e. 23.46384213
- f. 22.25713216

14. A parabola passes through the points (11.6, 14.3), (21.2, 71.3) and the point of intersection of the lines

$$27.6x - 341.5y = 131.2$$

and

$$-32.45x + 17.53y = 756.$$

Give the sum of the coordinates of the vertex of the parabola.

- a. -42.04020561
 - b. -43.06236533
 - c. -37.31145211
 - d. -44.63145527
 - e. -40.96783224
 - f. -41.36133621
15. Susan has total savings of \$2360 and Henry has total savings of \$2730. Henry and Susan combine their savings and spend \$113 of their savings on groceries. The remaining portion of their savings is invested in a fund that pays 7% per year compounded annually, with the amount at the end of each year rounded to the nearest penny. How much money will there be in the fund after 20 years?
- a. \$19,257.33
 - b. \$19,258.21
 - c. \$19,259.45
 - d. \$19,257.34
 - e. \$19,257.32
 - f. \$19,257.30

16. Every odd number from 1 to 100 is squared, and every even number from 1 to 100 is cubed. The result of the sum of calculations on the odd numbers is subtracted from the result of the sum of the calculations on the even numbers. Give this value.

- a. 12834311
- b. 12836340
- c. 12835353
- d. 12838350
- e. 12837332
- f. 12839361

17. A particle is initially located at $(1,1)$ and it moves to many other points in the xy plane. When the particle is at the point (x, y) , it moves next to the point

$$(\text{mod}_{100}(x + y), \text{mod}_{100}(x + 2y)).$$

How many unique points has the particle been located at after 200 moves?

- a. 150
- b. 200
- c. 175
- d. 186
- e. 142
- f. 193

18. A particle is initially located at $(1,1)$ and it moves to many other points in the xy plane. When the particle is at the point (x, y) , it moves next to the point

$$(\text{mod}_{100}(x + y), \text{mod}_{100}(x + 2y)).$$

What is largest sum of the coordinates of the point that is visited the most?

- a. 197
 - b. 193
 - c. 185
 - d. 172
 - e. 188
 - f. 194
19. For each natural number n from 1 to 100, the line

$$(n^2 - 14n + 33)x + (n^2 - 30n + 56)y = 89n - 44n^2 + 2n^3$$

is created. What is the area of the region bounded by the horizontal and vertical lines?

- a. 206
- b. 302
- c. 208
- d. 186
- e. 244
- f. 198

20. For each natural number n from 1 to 100, the line

$$(n^2 - 14n + 33)x + (n^2 - 30n + 56)y = 0$$

is created. How many unique lines are created?

- a. 100
- b. 96
- c. 94
- d. 92
- e. 93
- f. 98

21. For each natural number n from 1 to 100, the line

$$(n^2 - 14n + 33)x + (n^2 - 30n + 56)y = 0$$

is created. How many unique lines do not intersect the parabola

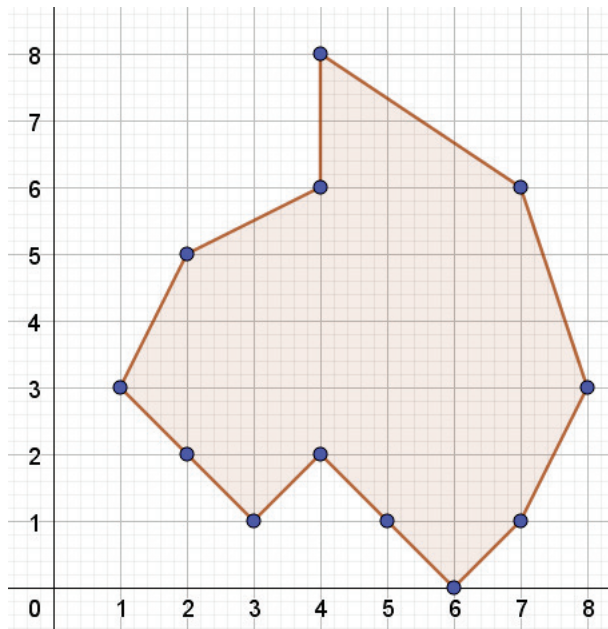
$$y = x^2 + 4?$$

- a. 90
- b. 91
- c. 94
- d. 93
- e. 89
- f. 92

22. A person is randomly given a number from 1 to 4. Then they roll a fair 6-sided die 20 times. After each roll, they add the amount rolled to their current number if the roll is even, and they subtract the amount rolled from their current number if the roll is odd. The result is their new number. Approximate the probability that their final number will be at least 20.

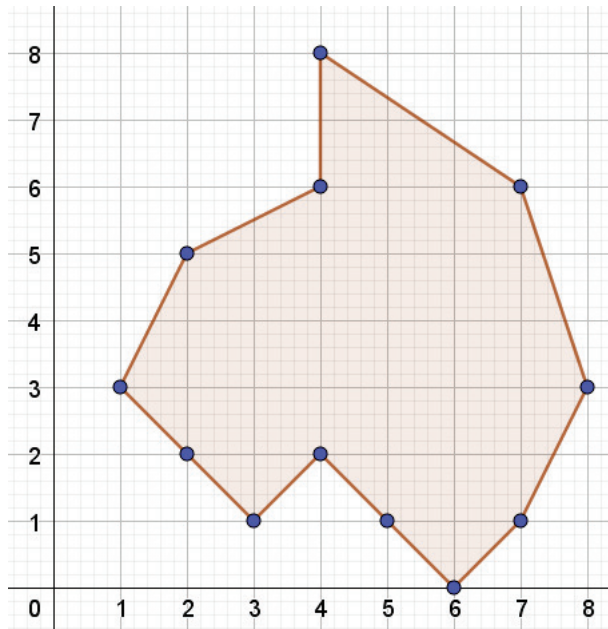
- a. 0.29
- b. 0.34
- c. 0.31
- d. 0.36
- e. 0.38
- f. 0.28

23. Give the area of the region shown below.



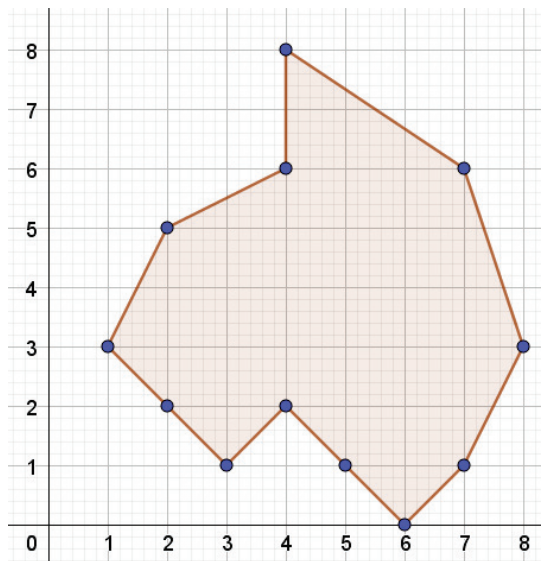
- a. 30
- b. 30.5
- c. 31
- d. 31.5
- e. 32
- f. 32.5

24. Give the sum of the coordinates of the centroid of the region shown below.



- a. 8.535519126
- b. 8.534216647
- c. 8.543122667
- d. 8.561241132
- e. 8.563533211
- f. 8.541564432

25. The region below is rotated 30° counterclockwise around the origin. Give the sum of the coordinates of the vertex in the rotated region that is furthest from the origin.



- a. 11.76318843
- b. 11.77422162
- c. 11.74326711
- d. 11.79314427
- e. 11.74766311
- f. 11.75833025

26. An integer point is a point of the form (x, y) where each of x and y are integers. How many integer points are contained strictly inside the circle of radius 81 centered at $(1.2, 3.4)$?

- a. 20054
- b. 20128
- c. 20423
- d. 20732
- e. 20931
- f. 20616

27. A prime point is a point of the form (x, y) where each of $|x|$ and $|y|$ are prime numbers. How many prime points are contained strictly inside the circle of radius 81 centered at $(1.2, 3.4)$?

- a. 1072
- b. 1383
- c. 1742
- d. 1639
- e. 1934
- f. 1808

28. Give the value of y associated with the solution to the system

$$\begin{aligned}11x + 2y + 7z &= 2 \\ -26x + 17y + 11z &= 21 \\ 27x - 3y + 43z &= 17\end{aligned}$$

- a. 0.325236372
- b. 0.125236372
- c. 0.425236372
- d. -0.125236372
- e. -0.225236372
- f. 0.925236372

29. Give the value of

$$\sum_{n=1}^{1000} \frac{1}{n^2 + n}.$$

- a. 0.888000888
- b. 0.777000777
- c. 1.099900099
- d. 0.999000999
- e. 1.077700077
- f. 1.088800088

30. A sequence of points is generated by setting $x_1 = 1$ and

$$x_{n+1} = (x_n)^2 - 1.2$$

for $n = 1, 2, 3, \dots$. Give the value of x_{20} .

- a. -0.214311621
- b. -0.351216637
- c. 0.241336812
- d. 0.166779184
- e. 0.184221663
- f. 0.214311621

31. A sequence of points is generated by setting $x_1 = 1$ and

$$x_{n+1} = (x_n)^2 - 1.5$$

for $n = 1, 2, 3, \dots$. Give the value of x_{2000} .

- a. -0.094916082
- b. -0.276916331
- c. -0.076915081
- d. 0.146888208
- e. 0.213665217
- f. -0.316643121

32. The interval $[-2,2)$ is divided into the 8 segments given by

$$S_m = [-2 + \frac{m}{2}, -2 + \frac{m+1}{2})$$

for $m = 0$ to 7 . A sequence of points is generated by setting $x_1 = 1$ and

$$x_{n+1} = (x_n)^2 - 1.5$$

for $n = 1, 2, 3, \dots, 2000$. Give the largest value of m so that S_m contains the largest number of values in this sequence.

- a. 6
- b. 4
- c. 5
- d. 7
- e. 1
- f. 3

33. The interval $[-2,2)$ is divided into the 8 segments given by

$$S_m = [-2 + \frac{m}{2}, -2 + \frac{m+1}{2})$$

for $m = 0$ to 7 . A sequence of points is generated by setting $x_1 = 1$ and

$$x_{n+1} = (x_n)^2 - 1.5$$

for $n = 1, 2, 3, \dots, 2000$. Give the largest value of m so that S_m contains the smallest number of values in this sequence.

- a. 6
- b. 7
- c. 4
- d. 3
- e. 2
- f. 0

34. A sequence of points is generated by setting $x_1 = 1$ and

$$x_{n+1} = \frac{1}{2}x_n + \frac{6}{2x_n}$$

for $n = 1, 2, 3, \dots, 20$. Find x_{20} .

- a. 2.454351662
- b. 2.454352723
- c. 2.449489743
- d. 2.454358112
- e. 2.337316653
- f. 2.453141133

35. Let $a > 1$. A sequence of points is generated by setting $x_1 = 1$ and

$$x_{n+1} = \frac{1}{2}x_n + \frac{a}{2x_n}$$

for $n = 1, 2, 3, \dots, 20$. Find the value a for which $x_{20} = 7.352$.

- a. 49.231663
- b. 54.136117
- c. 46.133893
- d. 55.321637
- e. 53.129871
- f. 54.051904