Precalculus Exam

University of Houston Math Contest 2025

1. If $f(2-x) = x - 1$ evaluate $f^{-1}(-2)$.

   (a) 0
   (b) -2
   (c) 3
   (d) -3
   (e) 2
   (f) None of the above.

2. Give the x-coordinate of the vertex of the parabola given by

$$y - x^2 = 4x - 4y.$$

   (a) 2
   (b) 0
   (c) 1
   (d) -1
   (e) -2
   (f) None of the above.

3. Give the exact value of

$$\sin\left(\tan^{-1}\left(\frac{-9}{40}\right) + \cos^{-1}\left(\frac{8}{17}\right)\right).$$

   (a) 512/554
   (b) 128/697
   (c) $-528/697$
   (d) 528/697
   (e) 1/2
   (f) None of the above.

4. $\sin^2(40^o) + \sin^2(50^o) =$

   (a) 1/2
   (b) 4/5
   (c) 3/5
   (d) 1
   (e) 2/3
   (f) None of the above.

5. If $f(x) = \sin(x)$ and $g(x) = \cos(x)$ what is $(f \circ g)(\frac{3\pi}{2}) + (g \circ f)(0)$?

   (a) 0
   (b) 1
   (c) 3
   (d) -1
   (e) 2
   (f) None of the above.

6. Simplify $\cos(x + 2\pi)\tan(x + \pi)$

   (a) $\sin(x)$
   (b) $\cot(x)$
   (c) $\cos(x)$
   (d) $\tan(x)$
   (e) $\cos(2x)$
   (f) None of the above.

7. Find the value of the expression $\sin(\tan^{-1}(\frac{1}{2}))$.

   (a) $\frac{\sqrt{5}}{5}$
   (b) $-\frac{\sqrt{5}}{5}$
   (c) 1
   (d) $\frac{1}{3}$
   (e) $\frac{1}{5}$
   (f) None of the above.

8. Evaluate the trigonometric expression

$$\frac{\cos^2(\frac{\pi}{5}) + \sin^2(\frac{11\pi}{5}) - 1}{\sin(\frac{\pi}{7})\sin(\frac{\pi}{6}) + 2}.$$

(a) 1
(b) 2
(c) 3
(d) 0
(e) 4
(f) None of the above.

9. If $\sin(13^\circ) = b$, what is $\cos(77^\circ)$ in terms of b?

(a) $\frac{1}{b}$
(b) $b\sqrt{2}$
(c) $-b$
(d) $90 - b$
(e) b
(f) None of the above.

10. If $\cot(y) = k$, what is $\cos(y)$ in terms of k?

(a) $\frac{1}{\sqrt{k^2+1}}$
(b) $\frac{k}{\sqrt{k^2+1}}$
(c) $\frac{1}{\sqrt{1-k^2}}$
(d) $\frac{k}{\sqrt{k^2-1}}$
(e) $2k$
(f) None of the above.

11. Which of the following is equivalent to the expression

$$2\cos(x) - \cos(90^o - x) + \sin(x) + \sin(90^o - x)?$$

(a) $3\cos(x)$
(b) $2\sin(x)$
(c) $2(\sin(x) + \cos(x))$
(d) 0
(e) 1
(f) None of the above.

12. Find the number of solutions to the equation $\sin(2x) = \frac{1}{2}$ for $0 < x < 2\pi$.

    (a) 1
    (b) 2
    (c) 3
    (d) 4
    (e) 5
    (f) None of the above.

13. Simplify
$$\frac{\tan(x) + \cot(x)}{\sec(x)\csc(x)}.$$

    (a) 1
    (b) $2\sin(x)$
    (c) $2\cos(x)$
    (d) $\frac{2}{5}$
    (e) 3
    (f) None of the above.

14. Give the period of the function $f(x) = 3\tan^5(2x - \pi/12) - 3$.

    (a) $\frac{15\pi}{2}$
    (b) $\frac{3\pi}{2}$
    (c) π
    (d) $\frac{\pi}{2}$
    (e) 6π
    (f) None of the above.

15. Find the solution(s) for the equation $\cos^2(x) + \sin(x) = 3/2$ on $[0, 2\pi)$.

    (a) $\frac{\pi}{4}, \frac{3\pi}{4}$
    (b) $\frac{\pi}{3}, \frac{7\pi}{6}$
    (c) $\frac{\pi}{4}, \frac{13\pi}{6}$
    (d) The equation has no real solutions.
    (e) $\frac{\pi}{6}, \frac{5\pi}{4}$
    (f) None of the above.

16. Two lines in the xy plane are perpendicular. One of these lines has positive slope and the angle between this line and the line $x = 4$ is $\pi/6$ radians. Give the sum of the slopes of the lines.

    (a) $2/\sqrt{3}$
    (b) $\sqrt{3}$
    (c) $-1/\sqrt{3}$
    (d) $-2\sqrt{3}$
    (e) $-\sqrt{3}$
    (f) None of the above.

17. Given $x + y + z = 180°$. Find $\cos(x)\cos(y) - \sin(x)\sin(y)$.

    (a) $-\sin(z)$
    (b) $\cos(z)$
    (c) $\sin(z)$
    (d) $-\cos(z)$
    (e) $\tan(z)$
    (f) None of the above.

18. Consider $\triangle ABC$ with $\angle ACB = 30°$, $BC = 6\ in$, and $AC = 4\ in$. Find the area of the triangle.

    (a) $3\ in^2$
    (b) $9\ in^2$
    (c) $12\ in^2$
    (d) $2\ in^2$
    (e) $6\ in^2$
    (f) None of the above.

19. To measure the height of a mountain, a surveyor takes two sightings of the peak from two different points on level ground, 1200 meters apart from each other, which are in line with the base of the mountain. The first observation of the peak results in a 30° angle of elevation, and the second observation results in a 60° angle of elevation. What is the approximate height of the mountain, in meters? (Disregard the height of the surveyor in your calculations.)

    (a) $600\sqrt{3}\,m$
    (b) $400\sqrt{3}\,m$
    (c) $500\sqrt{3}\,m$
    (d) $340\sqrt{3}\,m$
    (e) $300\sqrt{3}\,m$
    (f) None of the above.

20. The vectors $u = -5i + 5j$ and $v = xi + 2j$ are orthogonal. What is x?

    (a) -2
    (b) 2
    (c) 1
    (d) 8
    (e) -6
    (f) None of the above.

21. The vector $u = ai + bj$ is orthogonal to the line $y = bx - 2$. What is the largest possible value of $a + b$?

    (a) 1/2
    (b) 1/4
    (c) 1/3
    (d) 1
    (e) 2/5
    (f) None of the above.

22. Give the vertical asymptote to the graph of $f(x) = -\ln(15 - x^2 - 2x)$ corresponding to the smallest value of x.

(a) $x = -2$
(b) $x = 2$
(c) $x = -3$
(d) $x = -5$
(e) $x = 3$
(f) None of the above.

23. Give the domain of the function $g(x) = 6 + 3\ln(2x + 3)$.

(a) $(\frac{-3}{2}, \infty)$
(b) $(\frac{3}{2}, \infty)$
(c) All real numbers.
(d) $(-3, \infty)$
(e) $(1, 2)$
(f) None of the above.

24. Give the smallest solution to the equation

$$\ln x = \ln(x + 6) - \ln(x - 4).$$

(a) -1
(b) 1
(c) 3
(d) 2
(e) 6
(f) None of the above.

25. If $\vec{a} = \langle 1, 2\rangle$ and $\vec{b} = \langle 6, 9\rangle$, what is the the magnitude of the vector $\frac{1}{3}\vec{b} - \vec{a}$?

(a) $\sqrt{2}$
(b) 2
(c) $\sqrt{3}$
(d) $\sqrt{3}/2$
(e) 1
(f) None of the above.

26. If
$$y = \frac{a}{16}x - 1 \quad \text{and} \quad y = bx + 1$$
are equations of perpendicular lines, and $a + b = 6$, then what is the difference between the largest and smallest possible values of a?

(a) 11
(b) 10
(c) 12
(d) 10.5
(e) 11.5
(f) None of the above.

27. Find the number of points where the graphs of
$$x^2 + y^2 = 25 \quad \text{and} \quad \frac{x^2}{16} + \frac{y^2}{4} = 1$$
intersect.

(a) 2
(b) 3
(c) 1
(d) 0
(e) 4
(f) None of the above.

28. The graph of the equation $y^2 = 2|x| + 4$ is symmetric with respect to

(a) Only the y-axis and the origin.
(b) The x-axis, y-axis and the origin.
(c) Only the origin.
(d) Only the y-axis.
(e) Only the x-axis and the origin.
(f) None of the above.

29. One solution to the equation $x^2 - 10x + 14 = 0$ can be written as $x = 5 + \sqrt{k}$, where $k > 0$. What is the value of k?

(a) 10
(b) 11
(c) 9
(d) 44
(e) 3
(f) None of the above.

30. How many solutions does $8\cos^2(\theta) - 6 = 0$ have on $0 \leq \theta < 2\pi$?

(a) 3
(b) 2
(c) 0
(d) 4
(e) 1
(f) None of the above.

31. Which of the following is an equation of a circle in the xy-plane with center $(-2, 0)$ and an area of 49π?

(a) $x^2 - 4x + y^2 = 3$
(b) $x^2 + 4x + y^2 = 3$
(c) $x^2 + 4x + y^2 = 49$
(d) $x^2 + 4x + y^2 = 45$
(e) $x^2 + y^2 - 4y = 3$
(f) None of the above.

32. Identify the type of conic given by the equation

$$4x^2 + y^2 - 8x + 4y + 4 = 0.$$

(a) parabola
(b) circle
(c) hyperbola
(d) ellipse
(e) line
(f) None of the above.

33. Suppose $a > 1$. Give the largest possible value of $2\cos(x) + a\sin^2(x)$?

(a) $(a+1)/a$

(b) $(a^2+1)/a^2$

(c) $(2a+1)/a$

(d) $(2a+1)/a^2$

(e) $(a^2+1)/a$

(f) None of the above.

34. A circle in the xy-plane has equation $(x-3)^2 + (y-1)^2 = 25$. Which point below does not lie in the interior of the circle?

(a) $(7, -1)$

(b) $(0, 0)$

(c) $(5, 4)$

(d) $(-2, 3)$

(e) $(3, 5)$

(f) None of the above.

35. Find the radius of the circle $4x^2 - 24x + 4y^2 - 4y = 539$.

(a) 26

(b) 13

(c) 12

(d) $\sqrt{13}$

(e) $\sqrt{11}$

(f) None of the above.

36. $3\sin(x) - 4\sin^3(x) =$

(a) $\sin(3x)$

(b) $\cos(3x)$

(c) $\tan(3x)$

(d) $\cot(3x)$

(e) $\sin(2x)$

(f) None of the above.

37. Give the solution set to $\sin^{-1}(x^2 + x - 12) = \pi$.

(a) $\{-4, 3\}$
(b) $\{-5, 6\}$
(c) $\{-5, 3\}$
(d) $\{-5, 2\}$
(e) $\{-4, 2\}$
(f) None of the above.

38. Give the sum of the x coordinates associated with the vertical asymptotes of

$$y = \frac{x^2 + x - 12}{x^2 - x - 6}.$$

(a) -4
(b) -2
(c) 1
(d) 3
(e) There are no vertical asymptotes.
(f) None of the above.

39. $f(x) = 6x^5 + 7|x|x$ is:

(a) an even function
(b) neither an odd nor an even function
(c) an odd function
(d) a polynomial function when $x > -1$
(e) a polynomial function when $x < 1$
(f) None of the above.

40. A number u is a fixed point of $f(x)$ if and only if $f(u) = u$. Let $a > 1$ and define

$$f(x) = \begin{cases} ax, & x \leq \frac{1}{a} \\ \frac{a}{2a-1}(2 - x), & x > \frac{1}{a} \end{cases}.$$

Give the sum of the fixed points of $f(x)$.

(a) $3a/(2a - 1)$
(b) $2a/(3a - 1)$
(c) $(3a + 1)/2$
(d) $(2a + 1)/3$
(e) $(3a + 2)/3$
(f) None of the above.