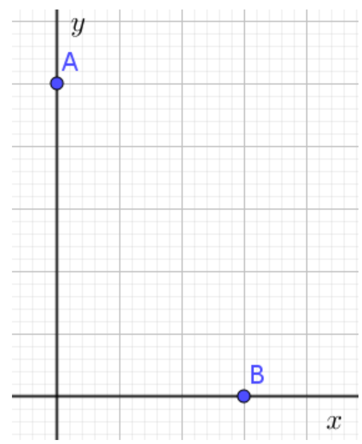
Algebra 2 Exam

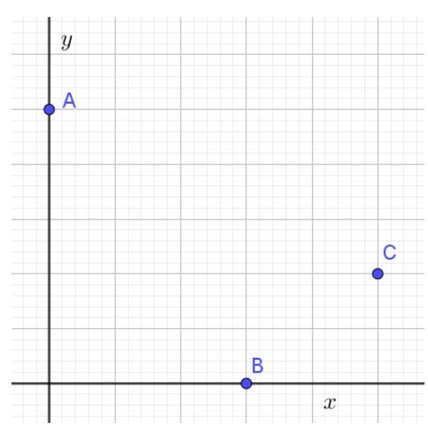
University of Houston Mathematics Contest 2026

1. The numbers $a, c, d > 0$ are integers and the roots of $x^2 + ax + 36 = 0$ are c and d. How many different choices are there for the value of a?

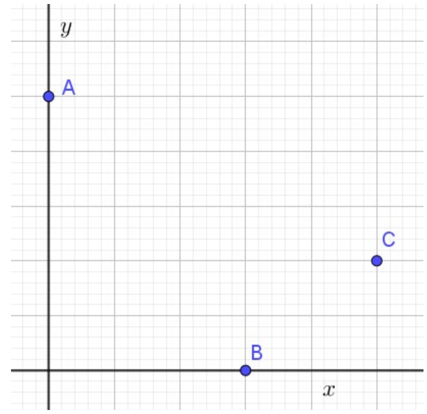
2. The major grid lines in the image are one unit each. A rectangle has sides parallel to the coordinate axes, a vertex at the origin, and its diagonally opposite vertex on the line segment connecting the points A and B. Give the maximum area of the rectangle.



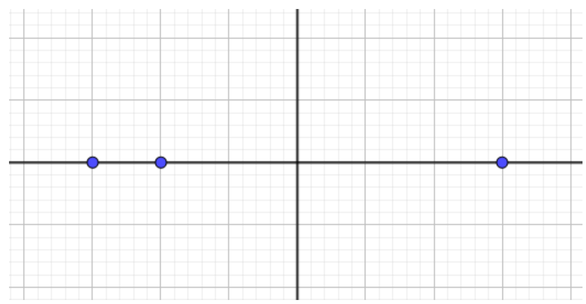
3. The major grid lines in the image are one unit each. $f(x)$ is a quadratic function whose graph passes through the points A, B and C. Give $f(1)$.



4. The major grid lines in the image are one unit each. $g(x) = a|x - b| + c$ and its graph passes through the points A, B and C. Find $a + b + c$.



5. The major grid lines in the image are one unit each. a, b and c are real numbers, and the x-intercepts of the function $f(x) = x^3 + ax^2 + bx + c$ are shown in the image. Give $f(0)$.



6. The number of a certain type of bacteria in thousands at time $t \geq 0$ (in minutes) is given by

$$A(t) = A_0 a^{rt}$$

where $A_0, a > 1$ and r is a real number. There are 2/3 as many bacteria in the culture 3 hours after it is created. If there were initially 60,000 bacteria in the culture, then how many hours after it is created will there be half as many bacteria in the culture?

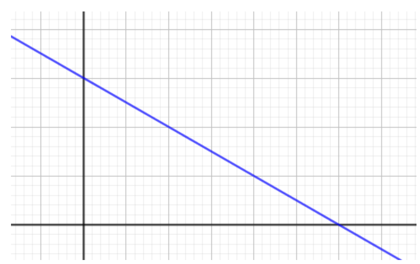
7. The major grid lines along the x-axis in the image are one unit each. A fifth-degree polynomial with leading coefficient -2 has the graph shown in the image and roots $1+i$ and $2-2i$. Give the sum of the coefficients of the polynomial.

8. Determine the number of solutions to $3^{\log_5\left(\log_2\left(\sqrt[13]{16x+4}\right)\right)}+1=0$.

9. The major grid lines in the image are one unit each. Give the sum of the values of a so that the line containing the points $(a,1)$ and $(1,b)$ is parallel to the line shown, and the system

$$x+2y-z=3$$
$$2x+ay+z=2$$
$$3x+y+bz=3$$

does not have a unique solution.

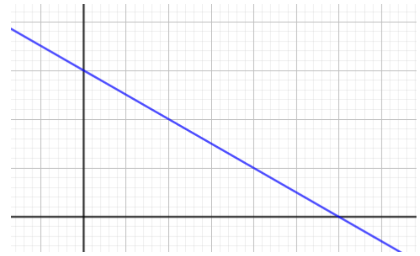


10. If a liters of a mixture that is 11% acid is mixed with b liters of a mixture which is 16% acid and 2 liters of a mixture that is 12% acid, it produces a solution that is 13% acid. However, when $3a$ liters of the mixture that is 11% acid is mixed with b liters of the mixture which is 16% acid and 2 liters of the mixture that is 12% acid, it produces a solution that is 12% acid. Give the sum of the values of a and b.

11. The major grid lines in the image are one unit each. Suppose a and b are real numbers. Find the value of $a + b$ for which the graph of

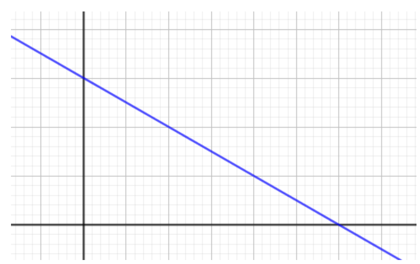
$$f(x) = (a + 2b + 1)x + 2a - 2b + 1$$

is perpendicular to the line shown and has x intercept 4.

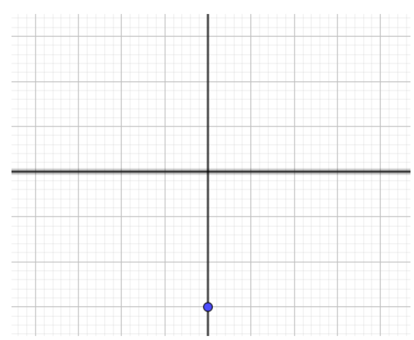


12. Give the largest number a so that the domain of $f(x) = \ln\left(3(2^x + 1) - \frac{1}{A}\right)$ is $(-1, \infty)$ when $a < A$.

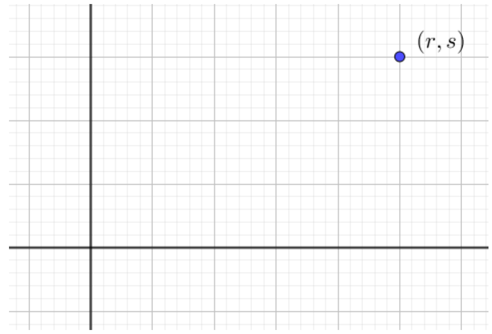
13. The major grid lines in the image are one unit each. The function $f(x) = \frac{1}{g(x)}$ where the graph of $g(x)$ is given in the image. Find $f^{-1}(2)$.



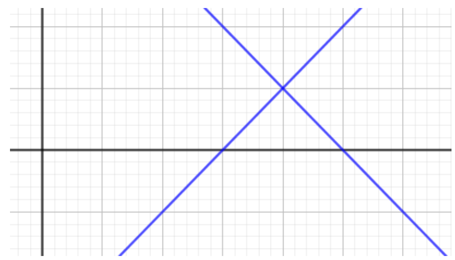
14. The major grid lines in the image are two units each. The function $f(x) = e^{2x} - 3e^x - a$ passes through the point shown in the image. Give the sum of the smallest value of $f(x)$ and the value x where the smallest value occurs.



15. The major grid lines in the image are one unit each. The variable A varies inversely as B^r, and B varies directly as C^s where the point (r, s) is shown in the graph. If A equals 32 when C equals 2, what is the value of A when C equals 4?



16. The major grid lines in the image are one unit each. The function $g(x) = 3x^2 + ax + b$ has roots that are identical to the roots of the lines in the image. Rewrite $g(x)$ in the form $g(x) = a(x-h)^2 + k$, and give the value of $a + h + k$.



17. Find the largest value c so that if $0 < b < c$ then the equation

$$\ln\left(\sqrt{12 - x^2}\right) = \ln(b) - \ln(x)$$

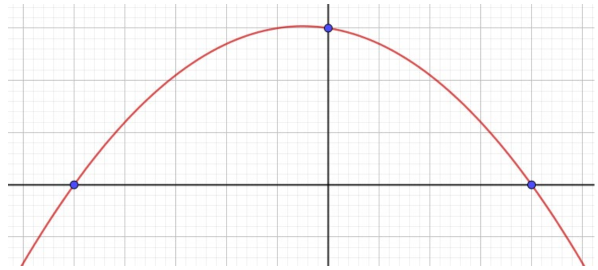
has two real solutions.

18. Give the sum of the values of the invertible functions below that can be evaluated at $x = 1$.

$f(x) = 2x^3 + x^4, g(x) = \ln(x+2), q(x) = \ln(x-2), h(x) = 2^{1-3x} + 6,$
$r(x) = x^2 + 3x + 1, R(x) = \sqrt{1-2x}, s(x) = 3x^3 + 2, F(x) = \sqrt[3]{3-x} + 2.$

19. Determine the smallest positive value of c so that the graphs of $f(x) = 3 - 2x$ and $g(x) = C|x-1| - C$ intersect at more than one point when $C > c$.

20. The major grid lines in the image are one unit each and the graph of the parabola $f(x)$ is shown. The domain of the function $g(x) = \sqrt{f(x)}$ is an interval of the form $[a, b]$. Find the maximum value of $g(x)$ on the interval $[a, b]$.

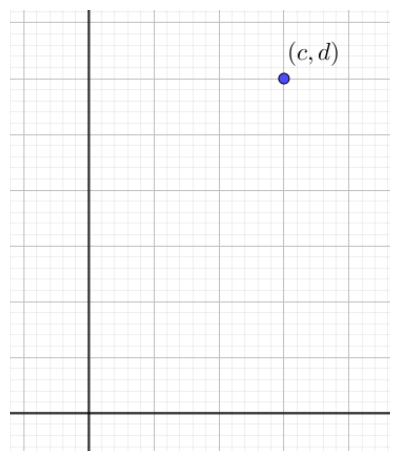


21. Let $f(x) = 3 - 2x$ and $g(x) = f^{-1}(x + 1)$. Find a and b so that the graph of $h(x) = x^2 + ax + b$ passes through the points $(g(2), 2)$ and $(2, g(2))$. Then give the smallest value c so that $h(x)$ is invertible for $x \geq c$.

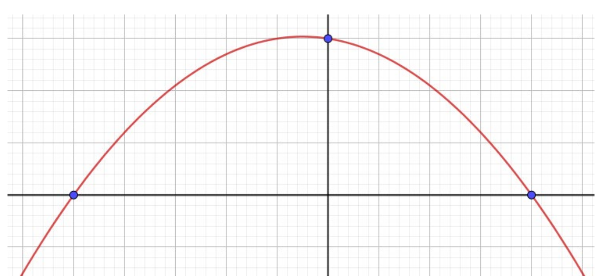
22. The major grid lines in the image are one unit each and the point (c, d) is shown in the image. Determine the values of a and b for which the system

$$\begin{pmatrix} ax + \dfrac{c}{y} = 5 \\ bx + \dfrac{d}{y} = 7 \end{pmatrix}$$

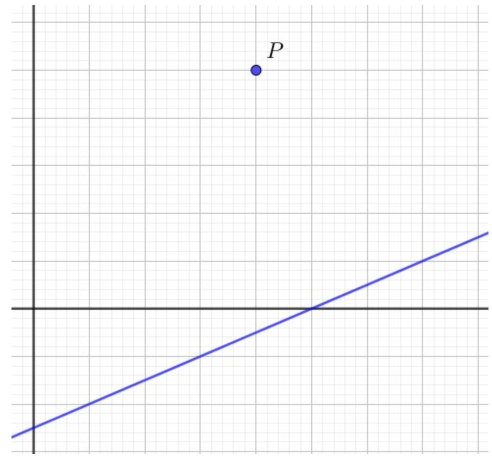
either does not have a solution or has more than one solution and then give $\frac{b}{a} + \frac{a}{b}$.



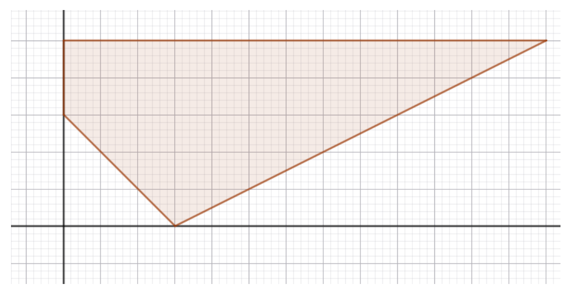
23. The major grid lines in the image are one unit each and the graph of the parabola $f(x)$ is shown. The domain of the function $g(x) = \sqrt{f(x)}$ is an interval of the form $[a, b]$. Give the slope of the line passing through the points $(3a, b)$ and $(3b, a)$.



24. The major grid lines in the image are two units each and the graph of $f(x)$ and the location of the point P are shown. Give the sum of the coordinates of the point that is a distance 4 from the point P and is closest to the graph of $f(x)$.



25. The major grid lines in the image are one unit each. Give the difference between the largest and smallest values of $4x - 3y$ for points (x, y) in the shaded portion of the image.



26. The major grid lines in the image are one unit each. a and b are numbers and $f(x) = x^3 + (4 + a)x^2 + bx - 2a$. The graph of $f(x)$ passes through the points shown in the image. Give the value of $f(a)$.