

Calculation Exam

University of Houston Mathematics Contest 2026

1. Let $S = \{1, 2, \dots, 100\}$. Give the average of the cubes of the numbers in S that are multiples of 3.
2. A sequence is generated by setting $x_1 = 2$,

$$\begin{aligned}x_2 &= \frac{1 - x_1}{3} \\x_3 &= \frac{1 - x_1 + x_2}{4} \\x_4 &= \frac{1 - x_1 + x_2 - x_3}{5} \\x_5 &= \frac{1 - x_1 + x_2 - x_3 + x_4}{6} \\&\vdots\end{aligned}$$

Give the value of x_{100} .

3. S is the set containing the smallest 243 natural numbers that are not divisible by any of 3, 5 or 11. Give the sum of the numbers that are in S .
4. The line $y = x$ divides the circle of radius 7 centered at $(3, 4)$ into two pieces. Give the area of the smallest piece.
5. Every even number from 1 to 100 is cubed, and every odd number from 1 to 100 is divided by 2. The result of the sum of calculations on the odd numbers is subtracted from the result of the sum of the calculations on the even numbers. Give this value.
6. Find the largest solution to the equation

$$\frac{2}{131}x^4 - 15x^3 - 12.7x^2 - 8x - 12 = 0.$$

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8. Give the slope of the line containing the points (101,213) and (317,412).

9. Give the distance from the point (72,63) to the line $13x - 41y = 11$.

10. Find the shortest distance from the circle $x^2 + 15x + y^2 + 11y = 42$ to the point (11,32).

11. Give the area of the triangle with vertices given by the intersections of the lines $12x - 13y = 16$, $4x + 3y = -12$ and $x + 3y = 16$.

12. Find the sum of the lengths of the line segments from the point (1,2) to the set

$$S = \{(\cos(2n), \sin(n)) : n = 1, 2, 3, \dots, 100 \text{ radians}\}.$$

13. Determine the set of all points (x, y) that solve the system

$$\begin{cases} x + 2y = 3 \\ 2x - y = t + 1 \end{cases}$$

for some real number t . What is the sum of the coordinates of the point in this set closest to $(-3, -2)$?

14. A number is represented in base 2 as 1100110011001100110011. Write this number in base 10.

15. Let $f(x) = 3x^2 - \frac{4}{2x-1} - 2.9$,
and define $g(x) = f(x^2)$. Give the smallest root of $g(x)$.

16. Give the acute angle of intersection (in radians) of the lines $31x + 23y = -12$ and $43x - 29y = 17$.

17. Give the average of the numbers

$$1, \left(\frac{2}{3}\right)^{-2}, \left(\frac{4}{5}\right)^{-2}, \left(\frac{6}{7}\right)^{-2}, \left(\frac{8}{9}\right)^{-2}, \dots, \left(\frac{100}{101}\right)^{-2}.$$

18. The vertices of a triangle are the centers of the circles $x^2 + x + y^2 + 2y = 6$, $2x^2 - 3x + 2y^2 - 4y = 7$ and $x^2 + y^2 - 5x - 7y = 2$. Give the area of the triangle.

19. Determine the number of points of intersection of the functions

$$f(x) = 43 \cos(17x - 21) + 10x$$

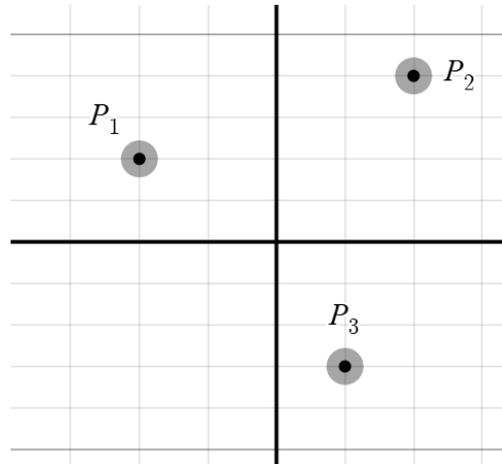
and

$$g(x) = 21 \sin(14 - 13x) - 5x.$$

20. An integer point has the form (a, b) where a and b are integers. Give the number of integer points that lie on or inside the circle of radius 87 centered at $(2, 3)$.

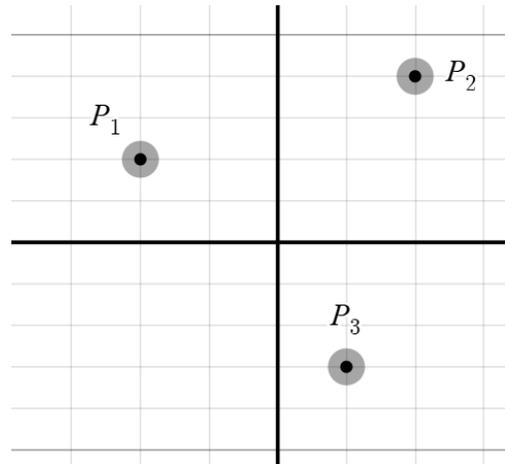
21. A particular type of steroid has a $\frac{1}{2}$ life of 4 hours inside a human body. As a result, if a person has $A(t)$ mg of the steroid in their body at time t (measured in hours), then there are $\frac{1}{2}A(t)$ mg in their body at time $t + 4$. Across a 48-hour period of time, a person ingests 8mg at time 0, 4mg at time 6, 4mg at time 12, 8mg at time 16, 4mg at time 24, 4mg at time 30, 4mg at time 36, and 8mg at time 40. Give the number of mg in the person's body at time 48.

22. The points P_1 , P_2 and P_3 are shown in the image below where the major hash marks are 1 unit in length. The line L_1 passes through P_1 and is perpendicular to the vector $-2i + j$, the line L_2 passes through P_2 and is perpendicular to the vector $i - 2j$, and the line L_3 passes through P_3 and is perpendicular to the vector $i + 3j$. Give the area of the region enclosed by these three lines.



23. A particle travels from the origin along 50 consecutive line segments that each have length 1. The i^{th} line segment is the line segment L_i from (x_{i-1}, y_{i-1}) to (x_i, y_i) for $i = 1, 2, \dots, 50$. The slope of L_i is 2 if i is even, and the slope of L_i is $1/2$ if i is odd. Give the value of y_{50} .

24. The points P_1 , P_2 and P_3 are shown in the image below where the major hash marks are 1 unit in length. Each of points P_i is the center of a circle C_i of radius 4. Give the sum of the coordinates of the integer points that lie outside the triangle $\Delta P_1 P_2 P_3$ and inside at least one of the circles.



25. The parents of a child invest \$5000 per year in a tax-free bond fund that earns 5% per year, compounded quarterly. All earnings are automatically reinvested in the bond fund. The first investment is made on the day the child is born, and the investments continue every year on the child's birthday up to and including their 17th birthday. If the child does not withdraw any money, how much money will be in the fund when the child is 50 years old? Give your answer to the nearest cent. Do not include a dollar sign. e.g. if your answer is \$643110.32 then give your answer as 643110.32.

26. If $f(x)$ is a given function, then a number u is a fixed point of $f(x)$ if and only if $f(u) = u$. Let

$$g(x) = \frac{10}{3}x(1-x)$$

and define $f(x) = g(g(x))$. Give the average of the fixed points of $f(x)$.

27. Let

$$f(x) = \frac{10}{3}x(1-x)$$

and $x_0 = 0.4$. Define $x_i = f(x_{i-1})$ for $i = 1, 2, \dots$. Give the average of the values $x_0, x_1, x_2, \dots, x_{1000}$.

28. Let $\theta_i = \frac{\pi}{2^{i-1}}$ for $i = 1, 2, 3, \dots$ and define

$$A_i = \begin{pmatrix} \cos(\theta_i) & -\sin(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i) \end{pmatrix} \text{ for } i = 1, 2, 3, \dots$$

Suppose \mathbb{N} is the set of natural numbers and $\theta = 3.6$. Define $\theta_{i_1}, \theta_{i_2}, \dots, \theta_{i_{30}}$ so that i_1 is the smallest natural number so that $\theta_{i_1} < \theta$, and for $j = 1, \dots, 30$, i_j is the smallest natural number so that $j > j - 1$ and

$$\sum_{k=1}^{j-1} \theta_{i_k} + \theta_{i_j} < \theta.$$

Define

$$A_\theta = \begin{pmatrix} \cos(\theta_{i_{30}}) & -\sin(\theta_{i_{30}}) \\ \sin(\theta_{i_{30}}) & \cos(\theta_{i_{30}}) \end{pmatrix} \begin{pmatrix} \cos(\theta_{i_{29}}) & -\sin(\theta_{i_{29}}) \\ \sin(\theta_{i_{29}}) & \cos(\theta_{i_{29}}) \end{pmatrix} \cdots \begin{pmatrix} \cos(\theta_{i_1}) & -\sin(\theta_{i_1}) \\ \sin(\theta_{i_1}) & \cos(\theta_{i_1}) \end{pmatrix}.$$

The matrix A_θ has the form

$$A_\theta = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

for some numbers a, b, c, d . Give the value of a .

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for some numbers a, b, c, d . Give the value of $c - \sin(\theta)$.