UNIVERSITY OF HOUSTON HIGH SCHOOL MATHEMATICS CONTEST CALCULUS, 2026

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Problem 1. Find the largest possible area for a rectangle with base on the x-axis and upper vertices on the curve $y = 1 - x^2$.

(A) $\frac{8\sqrt{3}}{9}$
(B) $\frac{4\sqrt{3}}{9}$
(C) $\frac{4}{3}$
(D) $\frac{2\sqrt{6}}{9}$
(E) $\frac{2\sqrt{3}}{9}$
(F) none of the above

Problem 2. Let f be the function with domain $(-\infty, \infty)$ given by

$$f(x) = 10\cos(9x).$$

Find the value of

$$f^{(4)}(0) + f^{(5)}(0) + f^{(6)}(0).$$

(A) $10 \cdot 9^4 - 10 \cdot 9^5 + 10 \cdot 9^6$
(B) $10 \cdot 9^4 + 10 \cdot 9^6$
(C) $-10 \cdot 9^4 + 10 \cdot 9^6$
(D) $10 \cdot 9^4 - 10 \cdot 9^6$
(E) $10 \cdot 9^4 + 10 \cdot 9^5 + 10 \cdot 9^6$
(F) none of the above

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Problem 3. What is the slope of the tangent line to the graph of the ellipse

$$x^2 + 2xy + 4y^2 = 12$$

at the point $(2, 1)$?

(A) $\frac{1}{2}$
(B) 2
(C) -2
(D) $-\frac{1}{2}$
(E) $\frac{1}{3}$
(F) none of the above

Problem 4. Evaluate the limit

$$\lim_{x \to 0} \frac{\sin(x^2)(5 - x)}{3x^2}.$$

(A) $\frac{5}{3}$
(B) $\frac{3}{5}$
(C) 1
(D) 0
(E) The limit does not exist as a real number.
(F) none of the above

Problem 5. Let $a > 0$ be constant. Evaluate

$$\int_0^a \frac{x}{a^2 + x^2}\, dx.$$

(A) a
(B) $\frac{\ln(a)}{2}$
(C) 1
(D) $\frac{\ln(2)}{2}$
(E) $\ln(2)$
(F) none of the above

Problem 6. The radioactive substance quantium decays exponentially. We have a sample of quantium of mass m_0 grams. The mass $m(t)$ of our sample at time t is given by $m(t) = m_0 e^{-kt}$, where time is measured in years and k is a positive constant. If the half-life of quantium is 50 years, how much of our sample is left after 75 years?

(A) $m_0 2^{-2/3}$ grams

(B) $m_0 e^{-2/3}$ grams

(C) $m_0 2^{-3/2}$ grams

(D) $m_0 e^{-3/2}$ grams

(E) $\frac{m_0}{3}$ grams

(F) none of the above

Problem 7. The surface area of a cube is increasing at a rate of 600 inches^2/hr. At what rate is the volume of the cube changing when each edge of the cube is 10 inches long?

(A) 1000 inches^3/hr

(B) 1200 inches^3/hr

(C) 1400 inches^3/hr

(D) 1500 inches^3/hr

(E) 1750 inches^3/hr

(F) none of the above

Problem 8. Define the function f with domain $(0, \infty)$ by

$$f(x) = x^{x^2}.$$

What is $f'(e)$?

(A) $e^{e^2}(4e)$

(B) $e^{e^2}(3e)$

(C) $e^{e^2}(2e)$

(D) e^{2e}

(E) $e^{e^2-1}(e^2)$

(F) none of the above

Problem 9. Define the function f with domain $(0, \infty)$ by

$$f(x) = \begin{cases} A\ln(x) + e^2, & \text{if } 0 < x < e, \\ B\cos\left(\frac{\pi x}{e}\right) + x^2, & \text{if } x \geqslant e, \end{cases}$$

where A and B are constants. There exist unique values for A and B that make f differentiable at $x = e$. If f is differentiable at $x = e$, what is the value of B?

(A) $-2e^2$
(B) $2e^2$
(C) πe^2
(D) $-\pi e^2$
(E) $-e^2$
(F) none of the above

Problem 10. A developing human organ is growing over time. The volume of the organ at time t is given by

$$G(t) = 40e^{-2e^{-t/10}},$$

where time is measured in months and volume in cm^3. At what time is the organ growing the fastest?

(A) 0 months
(B) $\ln(2)$ months
(C) $10 \cdot \ln(2)$ months
(D) 10 months
(E) $\frac{10}{\ln(2)}$ months
(F) none of the above

Problem 11. A string of length $L > 0$ is cut into two pieces. The first piece is used to form a square and the second piece is used to form a circle. Suppose we use γL of the string for the square and $(1 - \gamma)L$ of the string for the circle, where $0 < \gamma < 1$. What value of γ minimizes the total area enclosed by the square and the circle?

(A) $\frac{4}{\pi+4}$
(B) $\frac{1}{2}$
(C) $\frac{3}{4}$
(D) $\frac{2}{\pi+2}$
(E) $\frac{1}{\pi+1}$
(F) none of the above

Problem 12. The solid S has base given by the region in the xy-plane bounded by $x = 3 - y^2$ and $x = 0$. All cross sections of S obtained by slicing through S with planes perpendicular to the x-axis are squares. What is the volume of S?

(A) 9
(B) 18
(C) $\frac{9}{2}$
(D) 3
(E) $\frac{9\pi}{2}$
(F) none of the above

Problem 13. A ball is dropped from height H and falls in a vacuum until it hits the ground. Its position at time t is given by

$$z(t) = H - \frac{gt^2}{2},$$

where g is a positive constant. Here distance is measured in meters and time in seconds. What is the average velocity of the ball during its fall (averaging over time)?

(A) $-\sqrt{\frac{gH}{2}}$
(B) $-gH$
(C) $-\sqrt{gH}$
(D) $-2\sqrt{gH}$
(E) $-\frac{gH}{\sqrt{2}}$
(F) none of the above

Problem 14. Evaluate

$$\int_0^1 x\sqrt{1 - x^4}\, \mathrm{d}x.$$

(A) 1
(B) 2
(C) $\frac{\pi}{16}$
(D) $\frac{\pi}{8}$
(E) $\frac{\pi^2}{4}$
(F) none of the above

Problem 15. Evaluate

$$\lim_{r\to\infty}\lim_{x\to\infty}\left(\frac{[\ln(x)]^r}{x}+\frac{\sin(r)}{r}\right).$$

(You must evaluate the x-limit first and then the r-limit.)

(A) -1
(B) 1
(C) e
(D) $\frac{1}{e}$
(E) 0
(F) none of the above

Problem 16. Define the function F with domain all real numbers except zero by

$$F(x)=\int_0^{1+\frac{1}{x}} xt\,\mathrm{d}t.$$

What is

$$\lim_{x\to\infty} F'(x)?$$

(A) $\frac{1}{2}$
(B) $\frac{1}{4}$
(C) 0
(D) $-\frac{1}{4}$
(E) $-\frac{1}{2}$
(F) none of the above

Problem 17. Evaluate

$$\int_0^{\frac{\pi}{6}} \frac{\sin(\theta)+\sin(\theta)\tan^2(\theta)}{\sec^2(\theta)}\,\mathrm{d}\theta.$$

(A) $1-\frac{\sqrt{3}}{2}$
(B) $\frac{1}{2}$
(C) 1
(D) 2
(E) $1+\frac{2}{\sqrt{3}}$
(F) none of the above

Problem 18. The position of a particle moving on the real line is given at time t by $x(t)$, where $x(t)$ solves the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \tan^{-1}(3 - x^2).$$

For which positions x in $(-\infty, \infty)$ is the acceleration of the particle positive?

(A) $-\sqrt{3} < x < \sqrt{3}$
(B) $x < -\sqrt{3}$ and $x > \sqrt{3}$
(C) $-\sqrt{3} < x < 0$ and $x > \sqrt{3}$
(D) $x < -\sqrt{3}$ and $0 < x < \sqrt{3}$
(E) The acceleration is positive for no positions x in $(-\infty, \infty)$.
(F) none of the above

Problem 19. Let N be a positive integer. Suppose f is a differentiable function with domain $(-\infty, \infty)$ such that $f'(x) = 0$ for exactly N different values of x. What is the smallest of all positive integers S such that $f(x) = 0$ cannot be satisfied by at least S different values of x?

(A) $N + 4$
(B) $N + 3$
(C) $N + 2$
(D) $N + 1$
(E) N
(F) none of the above

Problem 20. Evaluate

$$\lim_{x \to \infty} (x^2 \tan^{-1}(x) + \sin(x)) \cdot \left(\frac{\mathrm{d}}{\mathrm{d}x} \tan^{-1} x \right).$$

(A) 1
(B) π
(C) $\frac{2}{\pi}$
(D) $\frac{\pi}{2}$
(E) $\frac{\pi}{4}$
(F) none of the above

Problem 21. Suppose f is a differentiable function with domain $(-\infty, \infty)$. Suppose that $f(3) = 5$ and suppose that there is a constant $M > 0$ such that $|f'(x)| \leqslant M$ for every real x. What is the largest value of M that guarantees $f(7) \leqslant 37$?

(A) 5
(B) 6
(C) 7
(D) 8
(E) 9
(F) none of the above

Problem 22. For which value of $a \in [0, \pi)$ is the equation

$$\lim_{n\to\infty} \left(\frac{\pi - a}{n}\right) \sum_{i=1}^{n} \sin\left(a + i\left(\frac{\pi - a}{n}\right)\right) = \frac{3}{2}$$

satisfied?

(A) 0
(B) $\frac{\pi}{6}$
(C) $\frac{\pi}{4}$
(D) $\frac{\pi}{3}$
(E) $\frac{2\pi}{3}$
(F) none of the above

Problem 23. Suppose a person begins training at time $t = 0$. Let $S(t)$ denote skill at time t, where $S(0) = 0$. Assume there exists a maximal skill level $L > 0$ such that $0 \leqslant S(t) < L$ for every $t \geqslant 0$. Suppose that skill evolves according to the differential equation

$$\frac{\mathrm{d}S}{\mathrm{d}t} = 6(L - S).$$

What is $\lim_{t\to\infty} S(t)$?

(A) L
(B) 0
(C) $\frac{L}{6}$
(D) $\frac{L}{3}$
(E) $\frac{L}{2}$
(F) none of the above

Problem 24. Define the function f with domain $(1, \infty)$ by $f(x) = \ln(\ln(x^2))$. Find $(f^{-1})'(\ln(2))$. Here f^{-1} is the inverse function.

(A) $\frac{1}{e}$
(B) $\frac{1}{e^2}$
(C) e^2
(D) e
(E) $\frac{1}{\ln(2)}$
(F) none of the above

Problem 25. The particular solution of the separable differential equation

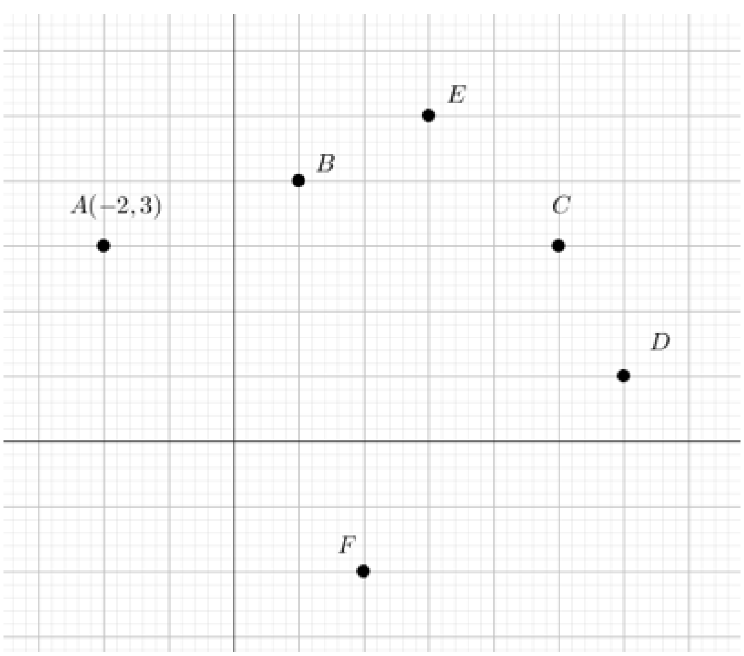
$$\frac{dx}{dt} = x^2$$

with initial condition $x(0) = 3$ satisfies $\lim_{t \to T^-} x(t) = \infty$ for some real number $T > 0$. What is T?

(A) 3
(B) $\frac{1}{3}$
(C) 1
(D) e
(E) $\frac{1}{e}$
(F) none of the above

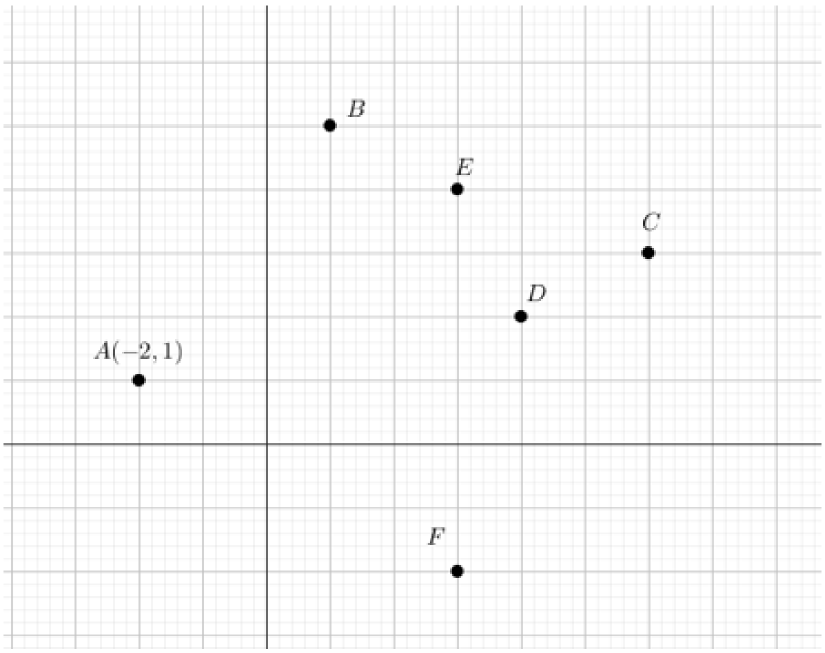
Problem 26. Refer to points in the diagram below as needed.

Consider the function $f(x) = 5x^2 + 6x + p$, where p is a constant. Find the slope of the tangent line to the curve at the point C marked on the diagram.



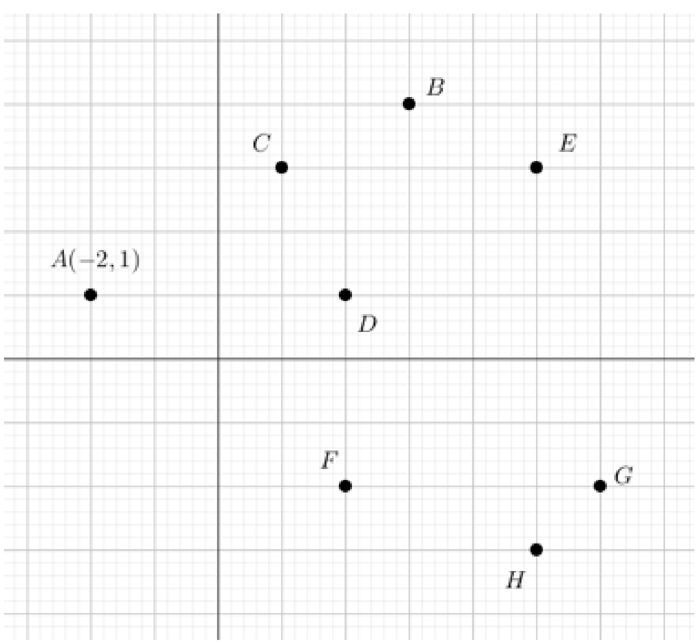
Problem 27. Refer to points in the diagram below as needed.

Given that $f(x)$ is an antiderivative of the function $g(x) = 3x^2 - 8x$. If the graph of $f(x)$ passes through the point D given on the diagram below, find $f(1)$.



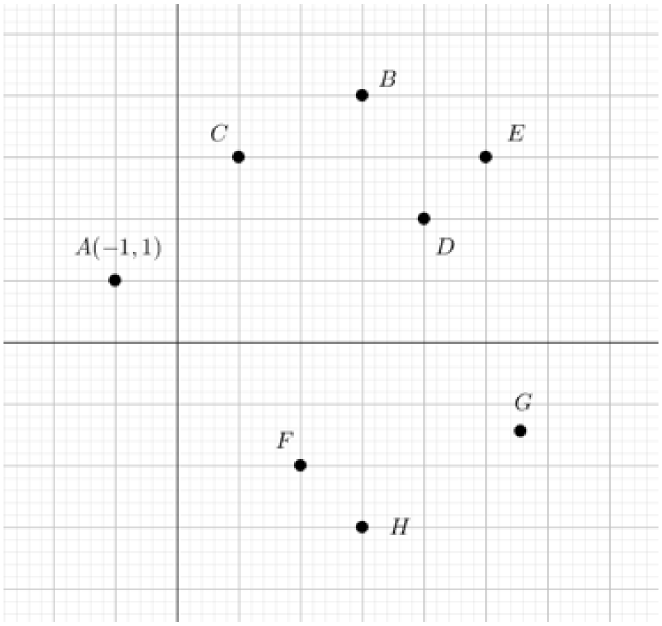
Problem 28. Refer to points in the diagram below as needed.

Consider the function $f(x) = px^2 - 30x + q$, where p and q are real numbers. If the function has a horizontal tangent line at the point E shown on the diagram, find the value of p.



Problem 29. Refer to points in the diagram below as needed.

Consider the function $f(x) = x^3 - Mx + N$, where M and N are real numbers. If the slope of the **normal line** to the function $f(x)$ at the point D is -1/2, find the value of M.



Problem 30. Refer to points in the diagram below as needed.

Given that a polynomial $f(x)$ has the following **first derivative**: $f'(x) = 2(x - m)(x - n)^3$, where m and n are real numbers. If the function $f(x)$ has local extreme points at the points C and E, find the value of $m + n$.