Precalculus Exam

University of Houston Mathematics Contest 2026

1. Let $f(x) = x^2 - 4x - 12$ be restricted to the domain $x \leq c$. If $f^{-1}(x)$ is well-defined and $f^{-1}(-12) = 0$, what is the maximum possible real value of c ?
   (a) -2
   (b) 0
   (c) 2
   (d) 4
   (e) 12
   (f) None of the above

2. If the graph of $f(x) = (2m - n)x^3 + x^{n+4} + 6x - 1$ is a parabola, then what is the value of $m + n$?
   (a) -3
   (b) -2
   (c) -1
   (d) 0
   (e) 2
   (f) None of the above

3. If $\cos\left(\frac{x}{2}\right) = a$ where $x < 90°$, which of the following is largest?
   (a) a
   (b) $\frac{1}{a}$
   (c) $\frac{1}{\sqrt{(1-a^2)}}$
   (d) $\sqrt{(1 - a^2)}$
   (e) $\frac{a}{\sqrt{(1-a^2)}}$
   (f) None of the above

4. If $f(x) = \tan(\pi x)$ and $g(x) = 2\sin(x)$, find the value of $f\big(g(\theta)\big)$ given that $\sin(\theta) = \frac{1}{4}$ and θ is acute.
   (a) 0
   (b) $\sqrt{3}$
   (c) 1
   (d) $\frac{\sqrt{3}}{2}$
   (e) Undefined
   (f) None of the above

5. Let x be an angle in Quadrant II. Which of the following statements must be true?
   (a) $\sin(x) + \cos(x) > 0$
   (b) $\tan(x) + \cot(x) > 0$
   (c) $\sin(2x) < 0$
   (d) $\cos(2x) > 0$
   (e) $\sec(x) > 0$
   (f) None of the above

6. A Ferris wheel with a radius of 25 feet is rotating at a constant rate. The center of the wheel is 30 feet above the ground. If a rider starts at the lowest point of the wheel at $t = 0$ seconds and completes one full rotation every 40 seconds, which of the following functions models the rider's height $h(t)$ in feet?
   (a) $h(t) = 25\sin\left(\frac{\pi}{20}t\right) + 30$
   (b) $h(t) = 25\cos(40t) + 5$
   (c) $h(t) = -30\cos\left(\frac{\pi}{20}t\right) + 25$
   (d) $h(t) = 25\sin\left(\frac{\pi}{40}t\right) + 5$
   (e) $h(t) = -25\cos\left(\frac{\pi}{20}t\right) + 30$
   (f) None of the above

7. Given that $\tan(\theta) = u$ and $\pi < \theta < \frac{3\pi}{2}$, find an expression for $\sec(\theta + 13\pi)$ in terms of u.

   (a) $\sqrt{(u^2+1)}$
   (b) $-\sqrt{(u^2+1)}$
   (c) $\frac{1}{\sqrt{(u^2+1)}}$
   (d) $-\frac{1}{\sqrt{(u^2+1)}}$
   (e) $\frac{u}{\sqrt{(u^2+1)}}$
   (f) None of the above

8. If $\vec{a} = 2\vec{\imath} - \vec{\jmath}$ and $\vec{b} = 2\vec{\imath} - 4\vec{\jmath}$, what is the magnitude of $5\vec{a} + \vec{b}$?

   Answer: __________

9. The range of the parabola $y = x^2 - 6x + k$ is $[-5, \infty)$. Find the value of k.

   Answer: __________

10. If $y = a$ is a horizontal asymptote of $f(x) = e^{x-2} + 1$, find $f(a)$.

    Answer: __________

11. A surveyor at point A measures the angle of elevation to the top of a tower to be $30°$. After walking 100 meters directly toward the tower to point B, the angle of elevation is $45°$. Find the height of the tower in meters.

    (a) $50(\sqrt{3}+1)$
    (b) $50(\sqrt{3}-1)$
    (c) $100(\sqrt{3}+1)$
    (d) $100(\sqrt{3}-1)$
    (e) $50\sqrt{3}$
    (f) None of the above

12. If $f(x) = \sqrt{(4-x)}$ and $g(x) = x^2 - 2$, find the domain of $f\big(g(x)\big)$.

(a) $(-\infty, \infty)$

(b) $(-\infty, 4]$

(c) $\left[-\sqrt{2}, \sqrt{2}\right]$

(d) $\left[-\sqrt{6}, \infty\right)$

(e) $\left[-\sqrt{6}, \sqrt{6}\right]$

(f) None of the above

13. Find the sum of all solutions to $\sin\left(2x + \frac{\pi}{4}\right) = \frac{1}{2}$ on the interval $[0, \pi]$.

(a) $\frac{\pi}{4}$

(b) $\frac{3\pi}{4}$

(c) $\frac{5\pi}{4}$

(d) π

(e) $\frac{7\pi}{8}$

(f) None of the above

14. Let $\sin(\alpha) = a$ and $\tan(\beta) = b$, where α and β are acute angles. Which of the following expressions is equivalent to $\cos(\alpha + \beta)$?

(a) $\frac{\sqrt{(1-a^2)}-a}{\sqrt{(1+b^2)}}$

(b) $\frac{\sqrt{(1-a^2)}-ab}{\sqrt{(1+b^2)}}$

(c) $\frac{\sqrt{(1-a^2)}+ab}{\sqrt{(1+b^2)}}$

(d) $\frac{a+b\sqrt{(1-a^2)}}{\sqrt{(1+b^2)}}$

(e) $\frac{a-b\sqrt{(1-a^2)}}{\sqrt{(1+b^2)}}$

(f) None of the above

15. The maximum value of $k\sin(x) + 12\cos(x)$ is 13. If $k > 0$, find the value of k.

Answer: __________

16. Given $\alpha + \beta = 90°$ and $0 < \beta < 45°$, which of the following is greatest?

(a) $\sin(\beta)$
(b) $\cos(\beta)$
(c) $\tan(\beta)$
(d) $\sec(\alpha)$
(e) All are equal
(f) None of the above

17. Which of the following is the sum of all possible values of k such that the vectors $\vec{u} = \langle k-1, 3 \rangle$ and $\vec{v} = \langle k, -2 \rangle$ are orthogonal?

Answer: __________

18. Find the sum of all real solutions to the equation: $\ln(x) + \ln(x+3) = \ln(18)$.

(a) -9
(b) -3
(c) 3
(d) 6
(e) 9
(f) None of the above

19. Let S be the set of all real solutions to the equation $4^x - 3 \cdot 2^{x+1} - 16 = 0$. Find the sum of the elements in S.

Answer: __________

20. If $f(3-x) = 2x + 1$, evaluate $f^{-1}(-3)$.

   (a) -3
   (b) -2
   (c) 1
   (d) 2
   (e) 5
   (f) None of the above

21. A rectangular board has a perimeter of 46 ft and an area of 120 ft^2. Find the length of the shorter side (width).

   Answer: ___________

22. Consider the equation $2x^2 + 8x + 2y^2 - 4y = k$. For which value of k is the graph of this conic section a single point?

   Answer: ___________

23. Find the sum of the solutions for $4\arctan(x^2 - 3x - 11) = -\pi$.

   (a) 3
   (b) -3
   (c) 4
   (d) 1
   (e) -1
   (f) None of the above

24. If $\cot(y) = k$, what is $\sin(y)$ in terms of k (where y is an acute angle)?

(a) $\frac{1}{\sqrt{(k^2+1)}}$

(b) $\frac{1}{\sqrt{(k^2-1)}}$

(c) $\frac{1}{\sqrt{(1-k^2)}}$

(d) $\frac{k}{\sqrt{(k^2+1)}}$

(e) $2k$

(f) None of the above

25. Evaluate the expression $\cos^2(A) + \cos^2(B)$ where $\angle A$ and $\angle B$ are two acute angles of a right triangle $\triangle ABC$.

(a) $\frac{1}{2}$

(b) $\frac{\sqrt{2}}{2}$

(c) 2

(d) 1

(e) 0

(f) None of the above

26. If the center of the ellipse $25x^2 + Bx + 4y^2 - 40y = -225$ is (h, k), where $h + k = 8$. Find the value of B .

Answer: __________

27. Find the direction angle of the vector $\vec{v} = -\sqrt{3}\vec{\imath} + \vec{\jmath}$

(a) $60°$

(b) $150°$

(c) $30°$

(d) $210°$

(e) $120°$

(f) None of the above

28. Let $f(x)$ be an even function and $g(x)$ be an odd function, both defined for all real numbers. Which of the following must be an odd function?

(a) $h(x) = f(x) \cdot g(x)$

(b) $h(x) = f(x) + g(x)$

(c) $h(x) = f\big(g(x)\big)$

(d) $h(x) = [g(x)]^2$

(e) $h(x) = f(x) - 5$

(f) None of the above

29. Given $0 < t < \frac{\pi}{2}$, simplify the following:

$$\frac{\sin\left(\frac{\pi}{2} - t\right)\sqrt{(1 + \tan^2(t))}}{\sqrt{(4\sec^2(t) - 4)}}$$

(a) $2\cot(t)$

(b) $\frac{1}{2}$

(c) $-\frac{\cot(t)}{2}$

(d) $\frac{\tan(t)}{2}$

(e) $\frac{\cot(t)}{2}$

(f) None of the above

30. Find the area of the circle defined by the equation $x^2 + y^2 - 6x + 14y + 33 = 0$.

(a) 5π

(b) 10π

(c) 25π

(d) 33π

(e) $5\pi\sqrt{2}$

(f) None of the above

31. Point $A(2,5)$ and point $B(k,-1)$ are endpoints of the diameter of a circle. If the line tangent to the circle at point B has a slope of $\frac{1}{2}$, what is the value of k ?

Answer: __________

32. Find the sum of all solutions to the equation $4\sin^2(\theta) - 3 = 0$ on the interval $[0,2\pi]$?

(a) 2π

(b) 3π

(c) 4π

(d) 5π

(e) 6π

(f) None of the above

33. Given a triangle $\triangle XYZ$, find $4\cos(X)\cos(Y) - 4\sin(X)\sin(Y)$.

(a) $4\cos(Z)$

(b) $-4\cos(Z)$

(c) $4\sin(Z)$

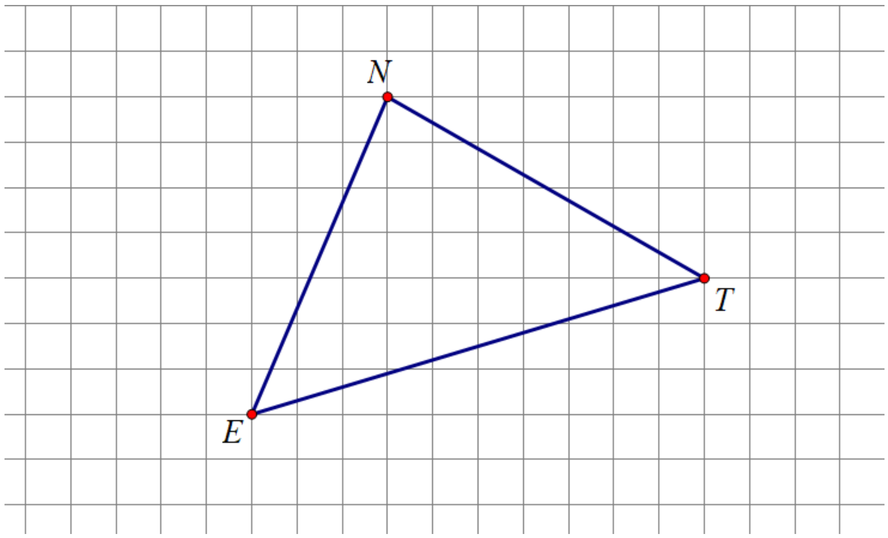
(d) $-\sin(Z)$

(e) $\cos(4Z)$

(f) None of the above

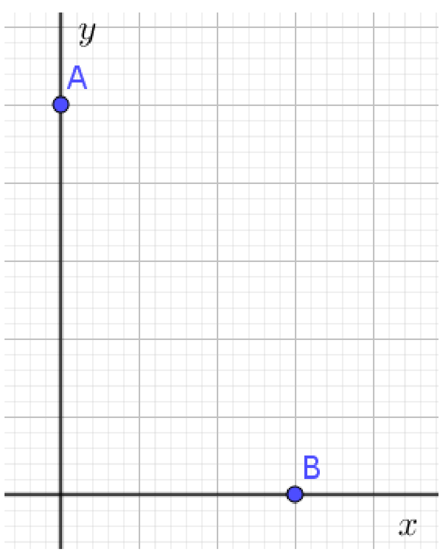
34. A triangle is shown below with vertices E, N, and $T(5,1)$.

Find the area of the triangle. (Grid lines are one unit each.)



Answer: __________

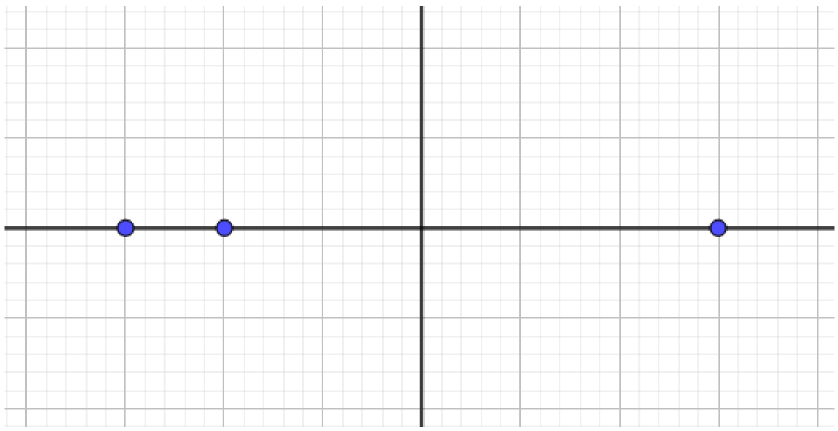
35. The major grid lines in the image are one unit each. A rectangle has sides parallel to the coordinate axes, a vertex at the origin, and its diagonally opposite vertex on the line segment connecting the points A and B. Give the maximum area of the rectangle.



Answer: __________

36. The major grid lines in the image are one unit each. a, b and c are real numbers, and the x-intercepts of the function

$f(x) = x^3 + ax^2 + bx + c$ are shown in the image. Give $f(0)$.

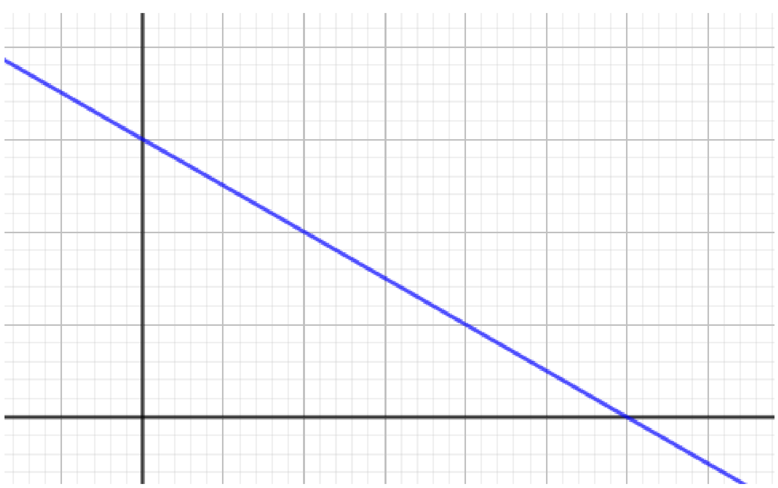


Answer: ___________

37. The major grid lines in the image are one unit each. Suppose a and b are real numbers. Find the value of $a + b$ for which the graph of

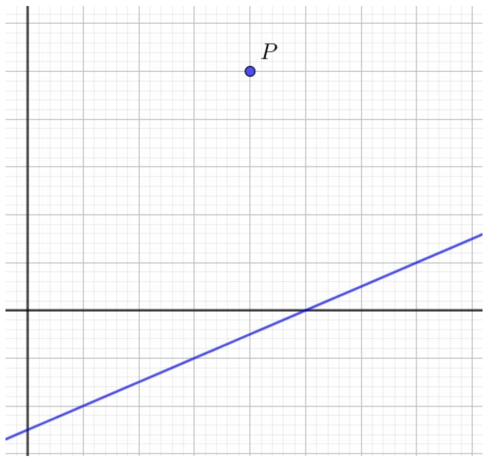
$$f(x) = (a + 2b + 1)x + 2a - 2b + 1$$

is perpendicular to the line shown and has x intercept 4.



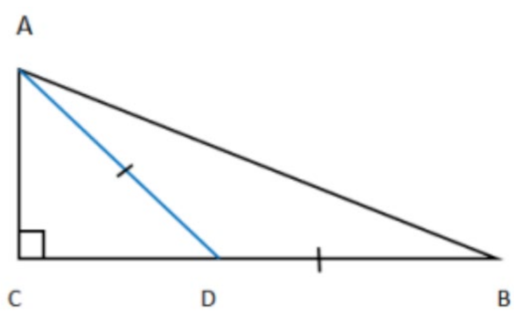
Answer: ___________

38. The major grid lines in the image are two units each and the graph of $f(x)$ and the location of the point P are shown. Give the sum of the coordinates of the point that is a distance 4 from the point P and is closest to the graph of $f(x)$.



Answer: ___________

39. Triangle $\triangle ABC$ with right angle $\angle C$ is shown below.



Given that $\sin(B) = \frac{1}{5}$ and $AD = BD$, find $\sin(\angle ADC)$.

Note: The diagram may not be drawn to scale.

(a) $\frac{24}{125}$

(b) $\frac{4\sqrt{6}}{25}$

(c) $\frac{23}{25}$

(d) $\frac{2\sqrt{6}}{25}$

(e) $\frac{2}{5}$

(f) None of the above