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University of Houston
High School Contest – Spring 2006
Precalculus Test

1. Evaluate and report your answer in simplest form: $\sqrt{\frac{\sin(\frac{5\pi}{6})\cos(\frac{3\pi}{4})\tan(\frac{5\pi}{4})}{\cos(\frac{2\pi}{3})\sin(\frac{7\pi}{4})}}$

2. The graph of $f(x)$ is the line through the points $(-3, 1)$ and $(5, 6)$. Determine the slope of the line that results from graphing $-f^{-1}(x)$.

3. Given $f(x) = 5x^3 + \frac{5}{x^3} - x - \frac{1}{x}$. Evaluate: $f\left(-\frac{1}{x}\right)$.

4. Given $f(x) = \frac{2x+1}{3x-2}$. Find $f^{-1}(x)$.

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5. Let $f(x) = x^2 - 4x - c$. Find a nonzero value for c so that $f(c) = c$.

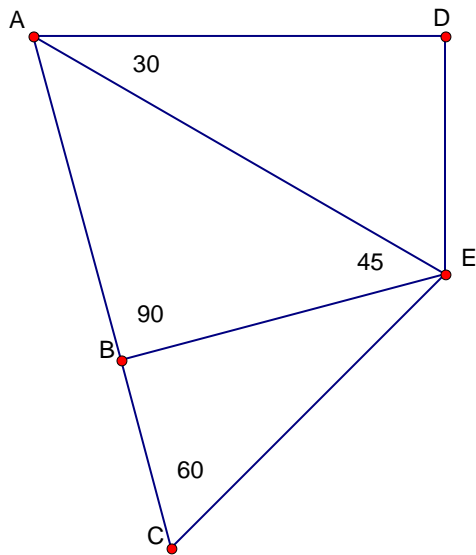
6. Find all solutions to $\frac{1}{1 + \frac{1}{1 + \frac{1}{x+1}}} = 2$

7. A parabola is tangent to the x axis, has an axis of symmetry of $x = 2$, and a y intercept of -4 . Give the equation for this parabola in standard form $f(x) = a(x - h)^2 + k$.

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8. Given the following figure, suppose that $\overline{AD} = 1$ and $m \angle ADE = 90$ and \overline{AC} is a straight line segment. Find the exact measure of \overline{CE} .



9. Determine the values of θ so that $0 \leq \theta \leq 2\pi$ and $\ln(\sqrt{1 - \sin \theta}) + \ln(\sqrt{1 + \sin \theta}) = \ln(\cos \theta)$.

10. Find the values of k for which the solutions to the equation $x^2 + 3x + k^2 = 0$ are real numbers.

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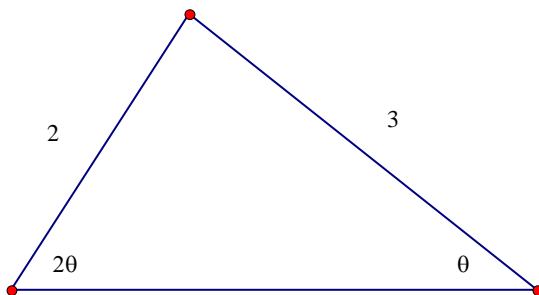
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11. Solve the inequality $\frac{2x+1}{x-1} - \frac{2}{x-3} < 1$.

12. Given $f(x) = \frac{3x}{x-2}$. The function $g(x)$ is $\frac{1}{x}$ after some shifting and reflecting, and $(f \circ g)(x) = -\frac{3x}{3x+2}$. What is $g(x)$?

13. Give the domain of the function $f(x) = \sqrt{\frac{\tan x - \frac{\sin^3 x}{\cos x}}{\tan x}}$, and then give $f(x)$ in simplified form.

14. Determine the measure of the angle θ in the following picture. Leave your answer in formula form.



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15. Which point on the curve $f(x) = \sqrt{x}$ is closest to the point $(1, 0)$?

16. Solve the equation $\sqrt{x^2 + 3x - 4} - \sqrt{x^2 - 5x + 4} = x - 1$.

17. Solve the equation $5\sqrt[3]{4 - x^2} = \sqrt[3]{(x + 2)^2} + 4\sqrt[3]{(2 - x)^2}$.

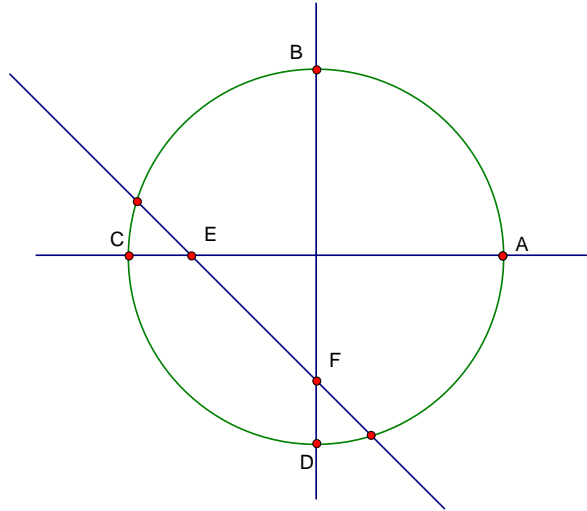
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18. What are the x coordinates of the points where the line intersects the circle?

The coordinates for the points in the illustration are:

- A (3, 0)
- B (0, 3)
- C (-3, 0)
- D (0, -3)
- E (-2, 0)
- F (0, -2)



19. Give the values for p and q so that the points $(-1, 1)$, $(-2, -5)$, $(2, 2)$, and (p, q) are the vertices of a parallelogram.

20. Give the largest value n so that n of the points

$$\left(1, \frac{1}{6}\right), \left(-2, \frac{7}{6}\right), \left(5, -\frac{7}{6}\right), \left(1, \frac{19}{6}\right), \left(-2, -\frac{5}{6}\right), \left(5, -\frac{13}{6}\right), \left(3, -\frac{5}{2}\right), \left(-1, -\frac{13}{6}\right)$$

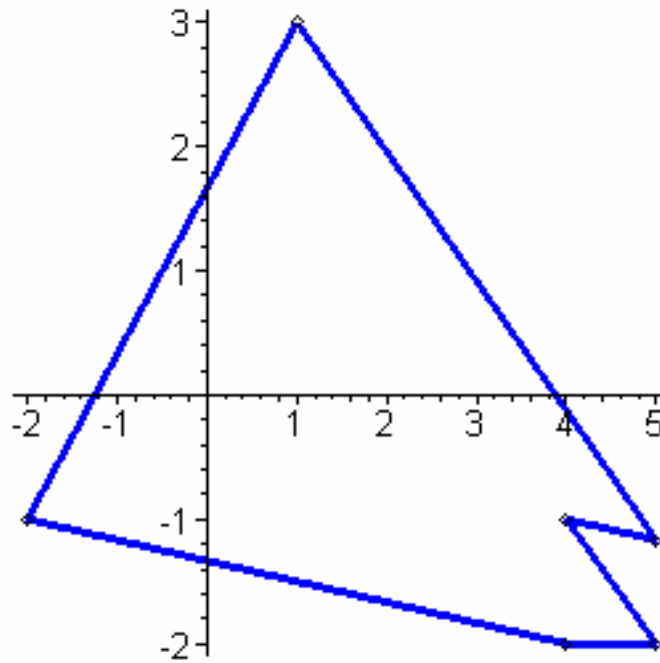
lie on the same line. Identify the points on this line and give the equation of the line.

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21. The set A consists of 10 distinct integers. The smallest of these integers is -6, and the largest is 5. The sum of the integers is 1. Give all possible choices for the set A .

22. It is well known that the area of a triangle with vertices $(a,b), (c,d), (e,f)$ is given by $\frac{1}{2} |(c-a)(f-b) - (d-b)(e-a)|$. Give the area of the polygon shown below.



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Answer Sheet - Precalculus Exam

Place your answers on this sheet. The work on your exam will only be graded in case of a tie breaker.

1.	12.
2.	13.
3.	14.
4.	15.
5.	16.
6.	17.
7.	18.
8.	19.
9.	20.
10.	21.
11.	22.